ON THE INTERTEMPORAL ELASTICITY OF SUBSTITUTION

by

Jonathan Adam Poeder

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Approved by:

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Dr. Benjamin Russo

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Dr. Jennifer Troyer

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Dr. Stanislav Radchenko

ABSTRACT

JONATHAN ADAM POEDER. On the Intertemporal Elasticity of Substitution.
(Under the direction of Dr. BENJAMIN RUSSO)

This thesis investigates the relationships among the average household's income evolution, consumption patterns, and labor supply decisions using data from the Panel Study of Income Dynamics. Demand studies such as this have, in the past, focused myopically on either labor supply or commodity demand. The conjoined approach taken here is novel in that it considers labor supply and commodity demand simultaneously. By using a Frisch framework for consumption and labor supply, along with estimating a simultaneous equation model, unbiased parameter estimates may be found. And that is the ultimate goal; the significance of which includes, but is not limited to, the ability to accurately calculate welfare changes resulting from tax policy adjustment.

ACKNOWLEDGEMENT

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PREFACE

For most freshmen the 'U' is simply the most coveted parking area on campus, but it means much more to me. It's the place I hugged my family for the last time and said goodbye before I would begin the most significant achievement of my life to date; earning two college degrees. I remember as they drove away and I walked toward Moore Hall, thinking to myself that a new and exciting part of my life was about to begin. I could never have imagined how great the experience would be, or how much UNCC would eventually mean to me.

That day marked the beginning of a journey where I would not only earn an education that I now hold dear, but find out more about who I am and who I want to be. I've learned much over the past eight years and I wouldn't trade the experiences or the people I've met for anything. But the time has come for me to move on, and although there is no place in economics for sentiment it is imperative that I express how important the experience of writing this thesis has been to me. It marks the culmination of my journey and as such I've poured my heart and soul into it for the past two and a half years.

This thesis has given me the opportunity to work with an amazingly intelligent and patient professor, who has given more to me in terms of knowledge and time than any other teacher I've had. I learned an incredible amount from him and will always be in his debt for what he has done for me. It is unlikely that I could have completed this endeavor without Dr. Russo's guidance. Finally, I would hope that my passion for economics and dedication to this work is evident in the pages that follow.

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CHAPTER 1: INTRODUCTION

Adam Smith's invisible hand theory from the Wealth of Nations says that by pursuing our own self interests we effectually do what is best for society as a whole. This thesis, in essence, attempts to identify certain details of that pursuit. More specifically, through empirical analysis we assess how individuals make consumption and labor supply decisions in a life-cycle setting. Historically, there has been plenty of work done in the area of consumer demand and labor supply economics. During the infancy of the literature the approach many economists used was logical, in a sense, but not sound. If one wanted to find the effect of changes in the wage rate on labor supply, for example, it made sense to regress the supply of labor on the wage rate and be done with it. But as we will see later on, the correct approach is much more complex. Labor economists were not the only ones, however, taking an incomplete approach to solving their problems as we find similar issues in the consumer demand literature as well.

In order to formulate a sound econometric model of labor supply or consumption it is necessary to take several factors into consideration. One of the most important things to remember when modeling labor supply and commodity demand is that either cannot be assumed separable from the other and therefore both must be estimated simultaneously.

And this is reasonable, since workers pay for daycare, travel costs, etc. oftentimes when working.

Another factor which should be considered is that models need to be dynamic in nature because people consider the future in their present day decisions. This issue will be discussed in more depth later, but in general people develop a consumption and labor supply 'profile' based on anticipated wages and future wage changes. In other words, in a two period life if an individual makes ten thousand dollars in period t but anticipates making one hundred thousand in period (t + 1), then it is reasonable to expect the average individual to consume more than ten thousand dollars worth of goods in period t and less than one hundred thousand in period two. This is, of course, if we make the reasonable assumption that credit markets work properly.

Again, this thesis attempts to identify, econometrically, how consumption and labor supply decisions are made in a life-cycle setting. The approach taken here is different in that the theoretical model and empirical specification take into consideration the aforementioned factors, like non-separability and the temporal nature of labor supply and consumer demand, which are so often disregarded. To begin, chapter two will consist of a literature review that will provide support for the methodology in this thesis. Chapter three will introduce the reader to what is known as the intertemporal elasticity of substitution, the estimates of which will be calculated in chapter six. Chapter four will discuss the data and the variables that have been included in our model of consumer demand and labor supply. In chapter five we will flesh out the empirical model and the methodology used to arrive at the results. Finally, chapter six will provide a discussion of the empirical results and the conclusions drawn from them.

CHAPTER 2: LITERATURE REVIEW

There has been much work done in the field of demand and labor supply economics at the macro level. However, the studies are consistently erroneous in the methodology undertaken leading to biased parameter estimates or even simple misinterpretation of the estimates themselves. Common mistakes include considering only static models, assuming separability of labor supply and commodity demand, and confounding the effect of wage changes on labor supply or commodity demand. (MaCurdy, 1981) This chapter will begin with an introduction to some principles of demand analysis from Deaton and Muellbauer's [D&M] *Economics and Consumer Behavior* (1980) that will prove useful to the reader later on. Subsequently, examples of labor supply and consumption from MaCurdy (1981) will be discussed followed by material supporting the methodology of this thesis including Abbot and Ashenfelter (1976), Heckman (1974), Browning and Meghir (1991), and Browning et al. (1985)

2.1. Demands & Constraints

When attempting to understand how changes in the after tax wage or interest rate affect labor supply and consumption decisions one may begin by employing a constrained optimization technique. This entails the use of an objective function, namely a utility or preference function, and a constraint. We assume a linear constraint in this thesis because implicit in this assumption is market efficiency and insignificant transactions costs (Deaton & Muellbauer, 1980). An example of a simple linear budget constraint can be written as follows:

$$x = \sum_{i=1}^{n} p_i q_i \tag{2.1}$$

Equation (2.1) says that total expenditure, x, is given by the sum of each good's price, p_i, multiplied by the quantity consumed, q_i. This constraint allows the individual to use all their purchasing power on a single good, but no more than the total expenditure given by x can be spent. The equality in (2.1) implies non-satiation, meaning the consumer always desires more than the budget constraint allows.

The basic representation of an uncompensated [Marshallian] demand function for good i is as follows:

$$q_i = f_i(x, p) \tag{2.2}$$

Here again x represents total expenditure and p is a price vector for all commodities. A general utility function is given by (2.3), which shows that utility is a function of the quantity of each good consumed. To derive (2.2) start by setting up the Lagrangian and differentiate with respect to the choice variables.

$$u = g(q_i) \tag{2.3}$$

This process just described should lead to a set of first order conditions, and is known as the Lagrangian multiplier method. It is not shown here because the procedure is standard and more useful as seen in later derivations. The first order conditions are then substituted back into the budget constraint, giving the uncompensated demand function in (2.2). The idea is to find the utility maximizing levels of the choice variables the individual demands subject to the budget limitations.

From (2.1) and (2.2) we get the adding up property of your standard demand function which can be written as:

$$x = \sum_{i=1}^{n} p_i f_i(x, p) \tag{2.4}$$

The interpretation of this is similar to that of equation (2.1) where f_i represents the Marshallian demand for good *i*.

Another property of demand functions is that of homogeneity. Uncompensated [Marshallian] demands are homogenous of degree zero in prices and expenditure, which means that a proportional increase in both prices and total expenditure leave the demand for good *i* unchanged. This can be written as

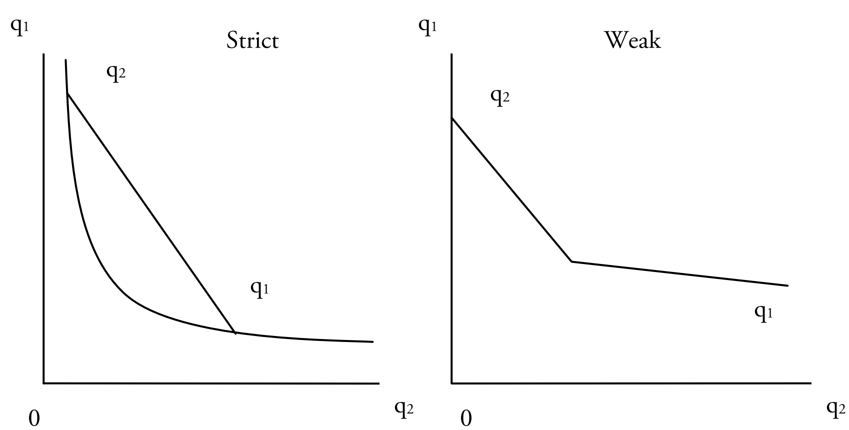
$$f_i(\gamma x, \gamma p) = f_i(x, p) = q_i; \text{ where } \gamma > 0 \tag{2.5}$$

The homogeneity restriction is also known as the absence of money illusion. (Deaton & Muellbauer, 1980)

The last property to be discussed is convexity, but we are referring to the convexity of a preference function as opposed to the demand function. With this there are two differing degrees that convexity can take, quasi or weak convexity and strict convexity. Intuitively, strict convexity says that any two combinations of goods along an indifference curve are preferred to those that lie along a lower indifference curve, which can be seen in Figure 2.1[1] below. Weak convexity is not as strong an assumption and basically does not guarantee diminishing marginal rate of substitution [DMRS] or that an individual is better off with a diverse combination of goods instead of all of one good and none of another.

The significance of this property may be unclear at first, but upon further inspection we find that the convexity of the preference set follows from the concavity or even quasi-concavity of the utility function. In other words, assumptions regarding concavity of the utility function yield strict convexity of preference sets and result in diminishing marginal rates of substitution.

Figure 2.1



2.2. Dynamic Labor Supply

Until fairly recent developments, models of labor supply have typically regressed hours of work on the wage rate and some measure of interest or 'unearned' income. MaCurdy (1981) develops a dynamic model of labor supply and employs the constrained

[1] Figure 2.1 is a reproduction of D&M figures on pages 29 and 30.

optimization technique described in section 2.1. The motivation for using a dynamic framework will be expanded upon in section 2.6, but these models are essentially necessary unless credit markets are completely imperfect and people are incapable of accumulating human capital.

As a result of employing a dynamic framework MaCurdy's model considers not only present day wages, wealth, and budget constraints in present day labor supply decisions, but future values as well. He starts with a general specification of a utility function [equation (2.6)], and assumes it to be strongly separable over time and monotonically increasing in its arguments. It follows from separability of the utility function that utility in one period is not dependent upon the value of the choice variables in another.

$$U = \sum_{t=0}^{T} \frac{1}{(1+\rho)^t} U[C(t), L(t)] \tag{2.6}$$

This function tells us that an individual's lifetime utility, U, is a function of the sum of discounted utility derived from consumption, C(t), and time away from work, L(t), over the course of the individual's life, the final period of which is given by T. The individual is assumed to live a life of $T + 1$ periods and utility is discounted using the individual's rate of time preference given by ρ.

Decisions based on equation (2.6) are subject to a budget constraint, which can be seen in equation (2.7).

$$A(0) + \sum_{t=0}^{T} R(t)N(t)W(t) = \sum_{t=0}^{T} R(t)C(t) \tag{2.7}$$

From (2.7), $A(0)$ represents the individuals stock of initial assets, $R(t)$ is a discount factor defined as $1/\{[1+r(1)][1+r(2)]\ldots[1+r(t)]\}$, $N(t)$ is the supply of labor, $W(t)$ is the wage rate, and T is again the final period of the individual's life. Equations (2.6) and (2.7) come directly from MaCurdy (1981) and using the Lagrangian multiplier method it is possible to arrive at the theoretical model of labor supply in his paper. The first order conditions are listed in equations (2.8) and (2.9), the derivation of which can be found in the Appendix A.

$$U_1[C(t),L(t)] = R(t)(1+\rho)^t \lambda, \qquad t = 0, \ldots, T \tag{2.8}$$

$$U_2[C(t),L(t)] \geq R(t)(1+\rho)^t \lambda W(t), \qquad t = 0, \ldots, T \tag{2.9}$$

Here λ is the Lagrange multiplier and is equal to the marginal utility of wealth in period 0. Equation (2.8) tells us that the marginal utility of consumption $[U_1]$ in a given period is equal to the marginal utility of wealth in period 0 after discounting. Equation (2.9) tells us that the marginal utility of time away from work $[U_2]$ in a given period, which is considered a 'good' in this context, is greater than or equal to the product of the wage rate and the marginal utility of wealth in period 0 after discounting.

The inequality in equation (2.9) allows for 'corner solutions', which means that the individual may or may not decide to supply labor in a given time period. In other words, if the marginal utility of time away from work is greater than the discounted product on the right side of the equation, the individual will not supply any labor. So, if the opportunity cost of working is greater than the product of the wage rate and the discounted marginal utility of wealth the individual will supply zero hours of labor.

Equations (2.8) and (2.9) along with the implicit function theorem, allow for the general representation of a theoretical model of dynamic labor supply. The implicit function theorem says that if 'y' is a function of 'x' and 'x' is a function of 'z', then 'y' is also a function of 'z'. It follows then that

$$N(t) = N[R(t)(1+\rho)^t \lambda, W(t)] \tag{2.10}$$

Equation (2.10) says that labor supply in period *t* [L^* - L(t) = N(t); L^* is the total time available to the individual] is a function of the discounted marginal utility of wealth and the wage rate in period *t*.

Equation (2.10) is a λ , or marginal utility of wealth, constant labor supply function, which in the literature has become known as a Frisch supply function. Frisch demand and supply functions will be discussed in more detail later. The optimal value of λ is determined using the budget constraint and is a function of what MaCurdy refers to as "consumer tastes", future and current wages, future and current interest rates, the rate of time preference, and initial assets (MaCurdy 1981). The significance of λ cannot be overstated, since it is only through λ that future factors considered in labor supply decisions at a point in time can affect those labor supply decisions. In other words, λ encompasses all previous, current and future information about wages and unearned income that is relevant to present day labor supply decisions. It is a sufficient statistic.

The next step is to develop an empirical specification of the utility function seen in equation (2.6), and then from it derive the labor supply function. The results of this process are given by (2.11) and (2.12), which represent a period specific utility function and the resulting labor supply function, respectively.

$$U_i[C_i(t), L_i(t)] = Y_{1i}(t)[C_i(t)]^{\omega_1} - Y_{2i}(t)[N_i(t)]^{\omega_2} \qquad (0 < \omega_1 < 1) \tag{2.11}$$

$$\ln N_i(t) = F_i + \delta \sum_{k=0}^{t} [\rho - r(k)] + \delta \ln W_i(t) + u_i(t) \qquad (\omega_2 > 1) \tag{2.12}$$

(2.11) and (2.12) come right from MaCurdy (1981), and the derivation of (2.12) can also be found in the Appendix A. Define δ as $\frac{1}{(\omega_2 - 1)}$, F_i as $\delta(\ln \lambda_i - \sigma_i - \ln \omega_2)$, and $\ln Y_{2i}$ as $\sigma_i - u_i^*(t)$. σ_i and $u_i(t)$ represent unmeasured characteristics of the individual with σ_i a permanent component and $u_i(t)$ a time varying component equal to $\delta u_i^*(t)$. Time varying and individual-specific modifiers of taste for consumption and labor supply are given by Y_{1i} and Y_{2i}, respectively. Lastly, time *invariant* parameters relating to consumption and labor supply that are similar among all individuals are given by ω_1 and ω_2, respectively.

Equation (2.12) marks the culmination of this section's purpose. This is MaCurdy's dynamic and estimable model of labor supply as a function of a time-invariant and individual specific component (F_i), the product of the intertemporal labor supply elasticity of substitution (δ) and the natural log of the wage rate at time *t*, the product of δ and the sum of the rate of time preference and interest rate differences, and a random component (u_i). "According to the estimates of the wage coefficients, δ lies in the range .10 - .23. The earnings coefficients indicate a range of .25 - .45 for δ." (MaCurdy, 1981, p. 1077) Also, "the real rate of interest exceeds the rate of time preference on average by about 2-4 percentage points" (MaCurdy, 1981, p. 1078). And, it is these estimates that are ultimately the source of interest in this research, this thesis notwithstanding.

MaCurdy's major contribution with this paper is the dynamic nature of the model, but unfortunately consumption and labor supply are implicitly assumed separable with the lack of simultaneous estimation. As will be discussed later, this is an erroneous assumption.

2.3. Dynamic Consumption

The intertemporal elasticity of substitution [*IES*] extends beyond labor supply analysis to demand analysis as well. The details of the intertemporal elasticity of substitution will be touched upon in the next chapter. As for now, an intuitive interpretation of the extension to consumption can be found in Browning (1989). Browning tells us that the higher the intertemporal consumption elasticity of substitution [IES_C], the more responsive are the movements of consumption to prices and interest rates from one period to another.

MaCurdy (1981) does not explicitly define or estimate a commodity demand function, although from the methodology prescribed it is possible to derive one as seen in equations (2.13) and (2.14) below. To arrive at them start again with equations (2.8) and (2.9) along with the implicit function theorem.

$$C(t) = C[R(t)(1+\rho)^t \lambda, W(t)] \tag{2.13}$$

$$\ln C_i(t) = V_i + \eta \sum_{k=0}^{t} [\rho - r(k)] + u_i(t) \tag{2.14}$$

Define η as $\frac{1}{(\omega_1 - 1)}$ and V_{i} as $\eta(\ln \lambda_i - \sigma_i - \ln \omega_1)$. This is a dynamic and estimable model of consumption as a function of a time-invariant and individual specific component (V_i), the product of the intertemporal consumption elasticity of substitution (η) and the sum of the rate of time preference and interest rate differences, and a random component (u_i). The process used to arrive at the demand functions in equations (2.13) and (2.14) is *exactly* the same as the procedure followed in 2.2 to formulate the theoretical and empirical specifications of the labor supply function in equations (2.10) and (2.12). The derivation of (2.14) can be found in Appendix A.

2.4. Separability

The topic of separability was mentioned in section 2.2 as a property of the specified theoretical utility function. In that context time separability, also known as time additivity, precludes the dependency of preferences in one period on consumption and leisure in another period. This is merely a simplifying assumption that has to do with the elimination of unnecessary complexities that can arise in a multi-period context. It is a reasonable assumption that allows for the tractability of labor supply and consumption functions.

Separability of the consumption and labor supply functions is another matter, and in fact Browning and Meghir (1991) test this assumption. The authors set up a conditional uncompensated demand system as the basis of the work in their paper. They start by dividing goods into two major categories including those that are of interest and those that are what they refer to as "conditioning goods" which include labor supply. Conditioning goods are not necessarily of particular interest themselves, but do have the ability to influence preferences over the goods that are of interest. The relationship may be clearer with an example, which can be seen in equation (2.15).

$$q_i = f(p, h, a, x \mid \theta_i) \tag{2.15}$$

This is the conditional uncompensated demand system from Browning and Meghir (1991), and it is readily evident that the quantity of good *i* is dependent on the standard price [*p*] and total expenditure [*x*] levels touched upon in section 2.1. In (2.15), however, we see that demand also depends on the quantity of conditioning goods demanded (given by *h*) and a vector of demographic variables (given by *a*). Lastly θ_i is a vector of parameters that allows for the same functional form of demand for each commodity, with varying sets of parameters.

This setup is advantageous for several reasons, one of which is that testing for separability is simplified. The test involves ascertaining the dependency of the demand system on the conditioning goods given by *h* on the right hand side of (2.15). Another advantage is that it is unnecessary to devise a specific preference construct for commodity demand and labor supply. Lastly, and maybe most significantly, there is not a need to model demand or the relevant constraints for the conditioning goods ultimately allowing the researcher to avoid calculating the optimal values for them. Browning and Meghir (1991) do mention a disadvantage of this approach, and it is that “all behavioral and policy implications are conditional on the quantities of conditioning goods consumed.” (Browning & Meghir, 1991, p. 931) So, for those purposes they must be calculated.

Within the context of this thesis, it is unnecessary to concern oneself with the aforementioned issue; the significance here is the accuracy of the separability assumption. Browning and Meghir run several Wald-type chi square tests of separability and find the test statistics for female and male labor supply are 31.49 and 61.77, respectively. The p-value for the female and male labor supply test statistics are .17% and .000001%, respectively, both of which are far less than the 1% significance level the researchers use in their tests. Therefore, one can reject the null hypothesis that commodity demand and labor supply are strongly separable.

2.5. Labor Supply & Commodity Demand Together

Abbot and Ashenfelter (1976) in their seminal paper "Labour Supply, Commodity Demand and the Allocation of Time", as the title suggests consider both labor supply and commodity demand in their estimation. It is a revolutionary approach that had not been taken previously in the labor supply or demand literature. The authors recognize how important estimates of pecuniary transfers made based on behavioral responses to tax and subsidy policies can be. The recognition of which is due in part to the findings of Becker (1965) and Mincer (1963) where they determine how the wage rate affects labor supply *and* commodity demand via the allocation of time and consumption decisions.

The approach Abbot and Ashenfelter take is clear and concise. First, they begin with a description of the theoretical framework including a general specification of the utility function (2.16) and the budget constraint (2.17) along with several restrictions that are imposed on the model.

$$u = u(1, x_1, \ldots, x_n) \tag{2.16}$$

$$\sum p_i x_i = w(T - l) + y = wh + y \tag{2.17}$$

Utility (2.16) is a function of leisure and commodities 1 through n while the budget constraint (2.17) gives total expenditure by summing the product of each commodity's price and quantity consumed. In addition, an individual's total expenditure must equal the sum of earned and unearned income given by wh and y, respectively. Earned income is calculated by multiplying the wage rate, w, by the difference in total time available T and leisure (l) one demands, h.

Abbot and Ashenfelter (1976) then use the constrained optimization technique mentioned previously while assuming fixed wages, prices, and unearned income to arrive at general representations of demand for commodities and leisure. Taken directly from their paper the equations are:

$$x_i = x_i(w, p_1, \ldots, p_n, y) \qquad i = 1, \ldots, n \tag{2.18}$$

$$l = l(w, p_1, \ldots, p_n, y) \tag{2.19}$$

Leisure, or more appropriately time away from work, is treated similarly to a market good where the wage rate represents the opportunity cost of time away from work. The amount demanded is hence a function of the wage rate along with the prices of each commodity given by p_i and unearned income as seen in (2.19). Commodity demand is also a function of all the aforementioned variables as seen in (2.18).

Some of the restrictions that the authors impose include that of homogeneity discussed previously, negative own-substitution effects, and symmetry restrictions as per Young's Theorem. Negative own- substitution effects refer to diminishing marginal utility in each of the commodities consumed including 'leisure' and the symmetry restriction guarantees that if there is a complementary relationship between a particular good and time away from work, then that good and labor supply are substitutes. With regards to the data, they use annually aggregated data from the US National Income and Product Accounts for the years of 1929-1967. It is collected for personal consumption expenditures on goods such as automobiles, food, and clothing along with wage rates and labor supply statistics.

Before estimation can begin, however, it is necessary to develop an estimable empirical specification of the aforementioned theoretical framework. Abbot and Ashenfelter (1976) provide three different empirical specifications of the utility function so as to be able to analyze the findings of each method for inconsistencies. However, it is unnecessary to detail each for the purpose of this thesis. The only one that will be discussed is the augmented Stone-Geary (1954) utility function, which takes the following form

$$u = B_0 \ln(l - \gamma_l) + \sum B_i \ln(x_i - \gamma_i) \qquad i = 1, \dots, n \tag{2.20}$$

From (2.20), $(x_i - \gamma_i) > 0$ $(i = 1, \dots, n)$ where γ_i represents the subsistence level of consumption of good i, $(l - \gamma_l) > 0$ where $\gamma_l$represents the subsistence level of time away from work, and $\sum B_i = 1$ $(i = 0, \dots, n)$. Finally, $T - \gamma_l \equiv \gamma_h$ is the maximum feasible working hours an individual may work.

Equation (2.20) is then assumed to be maximized subject to the budget constraint in (2.17) via the Lagrangian multiplier method, which yields the necessary first order conditions in equations (2.21) and (2.22)

$$B_i /(x_i - \gamma_i) = \lambda p_i \qquad i = 1, \ldots, n \tag{2.21}$$

$$B_0 /(l - \gamma_l) = \lambda w \tag{2.22}$$

Summing the $(n + 1)$ equations in (2.21) and (2.22) gives the solution for λ in equation (2.23), the derivation of which is in Appendix A.

$$\lambda = (wT + y - \sum p_i \gamma_i - w\gamma_l)^{-1} \qquad i = 1, \ldots, n \tag{2.23}$$

Using the solution for λ and the first order conditions it is possible to arrive at the expenditure functions for consumption and time away from work. These are given by equations (2.24) and (2.25).

$$p_i x_i = \gamma_i p_i + B_i (y + \gamma_h w - \sum \gamma_i p_i) \qquad i = 1, \ldots, n \tag{2.24}$$

$$-wh = -\gamma_h w + B_0 (y + \gamma_h w - \sum \gamma_i p_i) \qquad i = 1, \ldots, n \tag{2.25}$$

These 'expenditure' functions are essentially labor supply and commodity demand functions, and are estimated using maximum likelihood estimation.

For this approach, along with the two others, the authors estimate both the compensated and uncompensated price elasticities for commodity demand as well as labor supply. The difference between the two is that compensated elasticities are calculated holding the level of utility constant resulting in pure Hicksian substitution effects, while uncompensated elasticities are calculated holding the level of total expenditure constant resulting in both income and substitution effects. Both, however, simply describe the responsiveness of consumption and labor supply to changes in the prices and the wage rate, respectively.

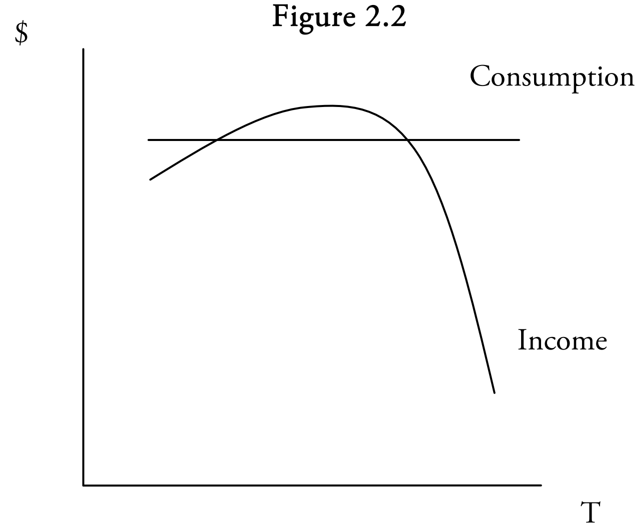
After completing the estimation for the three static approaches Abbot and Ashenfelter (1976) conclude that the incorporation of labor supply does not add any significant complexities to the analysis of consumer demand. There is some dissatisfaction with the standing of the research at the time and as such the authors call for further work to be done. This is not surprising since the model is static in nature, which will be shown in the next section to provide an inadequate framework. Nonetheless, this paper does provide a major contribution to the fields of labor supply and consumer demand in that both sides are considered.

2.6. Dynamic Models

So far we have mentioned some of the basics in the field of demand analysis, provided examples of labor supply and commodity demand, and discussed the importance of the intratemporal separability assumption. The final consideration when developing a model is its temporal nature. The purpose of this section is to familiarize the reader with the significance of dynamic modeling.

The life cycle hypothesis developed by Modigliani and Brumberg (1954) says that if the interest rate equals the rate of time preference consumption remains relatively constant over the life as is seen in Figure 2.2 below. Consumption patterns within the context of the life cycle hypothesis are ultimately determined, therefore, by the interaction of the rate of time preference and the interest rate but *not* contemporaneous income. The life-cycle

Figure 2.2

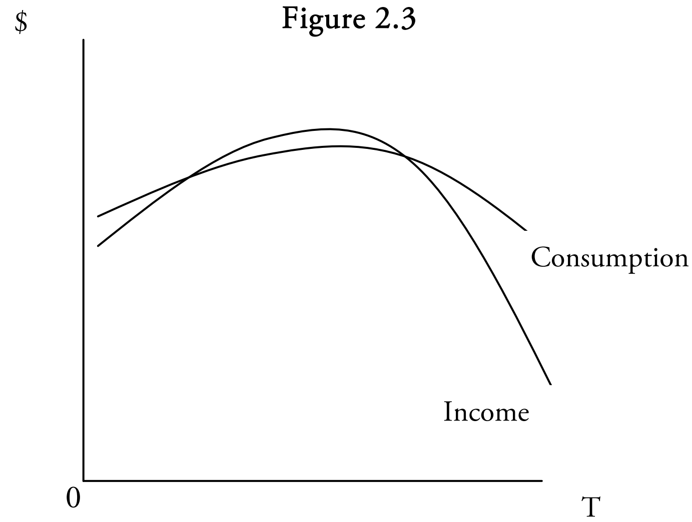


hypothesis is another seminal piece of work, but Thurow (1969) and Nagatani (1972) find empirical evidence that poses as a potential contradiction of the life-cycle hypothesis' prediction that there is not necessarily a relationship between income and consumption in a given period.

Thurow (1969) incorporates the assumption of imperfect credit markets while Nagatani (1972) imposes uncertainty, thus taking two different approaches in explaining the same fact – that there is indeed a relationship between age, income receipts, and consumption. Thurow "shows that income and consumption expenditure both peak in the age interval 45-54." (Heckman, 1974, p. 188) The middle-age peak and similarity in curvature can be seen in Figure 2.3.

Figure 2.3 indicates that early on in life individuals consume more on average than their income would normally permit. But in anticipation of higher future wages individuals borrow against that income so consumption can be 'smoothed' over the life. Later on in life

Figure 2.3



after retirement occurs and income drops off relatively quickly, individuals live off of accumulated wealth. This indicates a clear relationship between income receipts and consumption.

Heckman's (1974) approach does not employ restrictions like imperfect credit markets and uncertainty used by Thurow (1969) and Nagatani (1972), respectively, allowing for consumption profiles like those in Figure 2.3 with the use of a life-cycle theoretical framework. The major difference in Heckman's (1974) paper is that income is not exogenously determined. Wages are exogenous, varying systematically, but hours of work supplied are determined endogenously. The systematic variation of the wage rate allows for the individual to determine at t_0 in which periods more labor will be supplied. So, if leisure and consumption are substitutes for example, periods with higher wages will induce a greater supply of labor and hence more consumption over the life. The relationship of time away from work and consumption, whether they are substitutes or complements, determines the effect that higher wages will have on intra-period levels of consumption.

To demonstrate the direction of the relationship between time away from work and consumption, Heckman (1974) starts by using a constrained optimization technique similar to that employed in MaCurdy (1981). The objective function and budget constraint Heckman uses are listed in equations (2.26) and (2.27), respectively.

$$U = \int_0^T e^{-\rho t} U(L(t), X(t)) dt \tag{2.26}$$

$$A(0) + \int_0^T e^{-rt} \{W(t)[M - L(t)] - P(t)X(t)dt\} \tag{2.27}$$

Here *T* is the final period of the individual's life, *L(t)* is leisure, *X(t)* is consumption, ρ is the rate of time preference, r is the interest rate, *A(0)* is the initial stock of assets, *W(t)* is the wage rate, *M(t)* is the total time available to the individual, *L(t)* is time away from work, and *P(t)* is the price level. After setting up the Lagrangian and differentiating with respect to the choice variables found in the objective function, one arrives at the first order conditions listed in equations (2.28) and (2.29) seen below.

$$U_1(t) - \lambda e^{(\rho - r)t} W(t) = 0 \tag{2.28}$$

$$U_2(t) - \lambda e^{(\rho - r)t} P(t) = 0 \tag{2.29}$$

Equation (2.28) says that the marginal utility of time away from work is equal to the discounted product of the wage rate at time *t* and the marginal utility of wealth. Equation (2.29) says that the marginal utility of consumption is equal to the discounted product of the marginal utility of wealth and the price level.

Using the first order conditions and the implicit function theorem it is possible to formulate general specifications of λ-constant time away from work and commodity demand functions which are seen in equations (2.30) and (2.31).

$$L(t) = L[\lambda e^{(\rho - r)t} W(t), \lambda e^{(\rho - r)t} P(t)] \tag{2.30}$$

$$X(t) = X[\lambda e^{(\rho - r)t} W(t), \lambda e^{(\rho - r)t} P(t)] \tag{2.31}$$

The next step is to substitute the demand equations in (2.30) and (2.31) back into the first order conditions found in (2.28) and (2.29). Once this is done it is necessary to differentiate with respect to *W(t)* and *P(t).* This process yields equation (2.32) below.

$$\begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix} \begin{bmatrix} L_1 & L_2 \\ X_1 & X_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{2.32}$$

The first matrix on the left side of (2.32) is made up of four parts, the utility function differentiated twice with respect to $W(t)$ [U_{11}], the utility function differentiated twice with respect to consumption [U_{22}], and the utility function differentiated with respect to leisure and consumption [U_{21} and U_{12}]. Now, to prove that a relationship exists between consumption and income receipts one must show that L_1 and X_2 are not equal to zero. The last step is to multiply both sides of equation (2.32) by the inverse of the first matrix on the left hand side, after which we reach (2.33).

$$\begin{bmatrix} L_1 & L_2 \\ X_1 & X_2 \end{bmatrix} = \frac{1}{D} \begin{bmatrix} U_{22} & -U_{12} \\ -U_{21} & U_{11} \end{bmatrix} \tag{2.33}$$

D is defined as the determinant and it is readily evident from (2.33) that L_1 is equal to $\frac{U_{22}}{D}$ and X_2 is equal to $\frac{U_{11}}{D}$, and that both are in fact negative. This follows from the assumption that the utility function is strictly concave, which guarantees that $D > 0$ and both U_{11} and U_{22} are less than 0. A more detailed derivation of (2.28) - (2.31), and the final results can be found in Appendix A.

Before moving on to the next section three key points can be made based on the findings in Heckman (1974).

1. The life-cycle relationship between consumption and income receipts over time exists and it is not necessary to make assumptions regarding credit market imperfections or uncertainty to show it.
2. The framework provided allows the individual to develop a consumption profile similar to that seen in Figure 2.3. However, while theory is incapable of determining the relationship between leisure and consumption, they must be substitutes for Heckman's findings to hold true.
3. Although preferences and utility are intertemporally additive, the future is taken into consideration at t_0 when making consumption and labor supply decisions thus requiring the use of dynamic models.

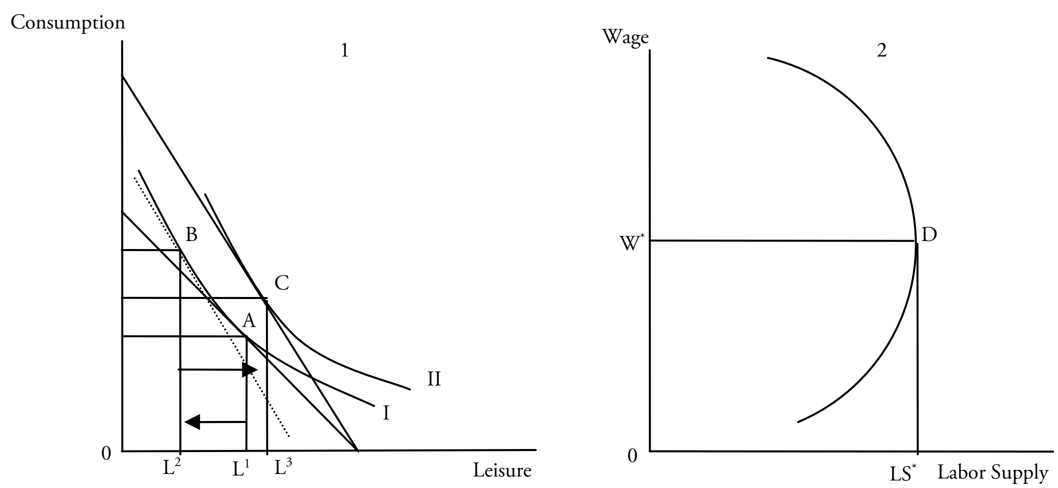
2.7. The Whole Picture…Almost

In 1985 Browning, Deaton and Irish published "A Profitable Approach to Labor Supply and Commodity Demands over the Life-Cycle", henceforth BDI (1985). In this paper the authors develop an ingenious framework for approaching labor supply and commodity demand modeling. The methodology is so clever, in fact, that it serves as the foundation for the steps taken in this thesis. There are shortcomings of course, which will be improved upon and discussed in more depth later on.

Sections 2.1 – 2.6 above present several aspects of the demand and labor supply literature that are integral to attaining a coherent picture of what a tractable empirical specification of the life-cycle hypothesis should include. BDI (1985) overcome the problematic aspects of other works in the field like assumed separability of the objective functions or a static temporal nature and provide a framework that is congruent with what has been previously deemed here as necessary. This section details that framework.

BDI begin by discussing previous labor supply analyses done in Great Britain and the fact that there are some contradictions in the results. Typically the models result in backward bending labor supply curves with zero or positive income effects. Theoretical presupposition associated with backward bending labor supply curves, however, mandates that income effects be negative and larger at some point than the positive substitution effect of a wage change. So, for the British studies to have backward bending labor supply curves and zero or positive income effects the substitution effect must be negative and larger than the income effect. This is a theoretical violation as the substitution effect must *always* be positive. A correct visual representation of how backward bending labor supply curves occur as a result of *negative* income effects can be seen in Figure 2.4. Graph 1 shows how an increase in the wage rate results in the substitution away from leisure, which is given by a movement along indifference curve I from point A to point B. However, the income effect that causes the jump from indifference curve I to II exceeds the substitution effect and moves the individual to point C. Conceptually, the individual reaches a level of income where the value of additional leisure exceeds the increase in the opportunity cost as measured by the change in the wage rate. Therefore increasing the wage rate induces a reduction in labor supply; this is the point where income effects exceed substitution effects and is denoted by the letter 'D' in graph 2.

Figure 2.4



As was done by Macurdy (1981) and Heckman (1974), BDI (1985) develop a model of consumption and labor supply that employs the Frisch framework. The ingenuity of their technique mentioned earlier lies in the use of a profit function where the differentiation of said function yields the Frisch demand and supply of interest. They provide elaborate details regarding the formulation of their model, but since it serves as the foundation for the approach taken in this thesis we will detail it in chapter five, 'The Model'. There we will discuss not only the formulation of the model, but the significance and importance of Frisch functions.

For now, marginal utility of lifetime wealth constant [Frisch] labor supply and commodity demand functions allow the researcher to separate the effects of anticipated and unanticipated wage changes. The separation is possible because 1) lifetime wealth and utility are held constant under Frisch demands and the life-cycle hypothesis and 2) the Lagrange multiplier [the marginal utility of lifetime wealth] in the standard maximization problem is a sufficient statistic for all past, present, and future information affecting the labor supply decisions made at the planning horizon's nascent. Additionally, as a result of 1 and 2, information regarding the shape of the labor supply profile can be found in the *IES*, which is derived from the wage rate's parameter estimate.

In the types of studies done by BDI, MaCurdy, and Heckman true panel data is of great value because it captures valuable individual heterogeneity otherwise lost in cross-sectional or time series data. Unfortunately true panel studies that gather data on both labor and consumption dynamics are currently impossible to come by. BDI provide a work around for this problem however by 'creating' panel data from the British Family Expenditure Survey. So, instead of tracking individual families they generate synthetic cohorts, grouped by age, and effectively treat each group as a single individual. The applicability of this technique to their approach lies in the fact that the price of lifetime utility[2] is assumed to be constant for each individual over the lifecycle. If this is the case then the cohort mean should be constant as well. In the uncertainty case any would be issues are dealt with by the first differencing of each process.

The first round of estimation assumes separability of the labor supply and commodity demand functions. In other words BDI assume goods and labor are additive

[2] Note that this term will also be developed more fully in chapter 5.

within periods so that $(\theta_1 = \theta_2 = 0)$, where θ is the coefficient on the wage-price ratio that links the two objective functions. They find that when manual and non-manual workers are delineated within the estimation of labor supply, cohorts and children leave little in terms of explanatory power for wages. When the two classes of workers are pooled they find that the wage coefficient becomes significant with the introduction of year dummies. The statistical significance of the year dummies implies that "behavior over the business cycle is not explicable in terms of life-cycle intertemporal substitution." (Browning, Deaton, & Irish, 1985, p. 532)

Next they estimate the labor supply regression under the assumptions of uncertainty and separability of labor and consumption and obtain results that are again not particularly supportive of the life-cycle hypothesis. Since there is the possibility of an endogeneity problem with the aforementioned assumptions, the change in the natural log of wages is instrumented with age, age squared, one period lagged price, and one period lagged wages. They are, however, estimated using OLS without instrumentation as well. Both methods indicate a negative relationship between changes in the wage rate and changes in hours.

Ultimately they find that under separability there is merely a simple positive correlation between wages and hours supplied across cohorts, but that this stands in sharp contrast to the negative relationship *within* cohorts. (Browning, Deaton, & Irish, 1985) This and the weakness of the estimates drive their opinion that the results have provided little evidence in favor of the theory. It turns out that the same holds true under the non-separability assumption when time dummies are excluded from the regression. When time dummies are included the evidence is somewhat in favor of the theory. Wage coefficients become significant and we find, as indicated by the price-wage ratio, that there is a complementary relationship between leisure and goods consumed. The positivity of these results is limited by the fact that the year dummies 'absolve' the wage coefficients from explaining the year to year fluctuations in hours. (Browning, Deaton, & Irish, 1985)

Their results are not limited to the labor supply regression. BDI note that prices do not follow any life-cycle pattern but that nevertheless they enter the demand side of the model with a negative sign and are nearly significant with the inclusion of children. (Browning, Deaton, & Irish, 1985) One inconsistency, however, is that the wage-price ratio is significant and *positive*, which of course indicates that goods and leisure are substitutes thus contradicting the results of the labor regression and the findings of Heckman (1974).

In sum, as is probably evident, their results are not in favor of the theory. The individual statistics are insignificant in most cases, children seem to be better determinants of labor supply, and there is a discrepancy among what exactly the relationship is between leisure and goods. The story does not stop there though. There are major econometric issues that need to be dealt with in their approach. The theoretical foundation is sound, but one of the most evident shortcomings is the loss of individual heterogeneity and its effect on the consistency of estimates from their use of synthetic cohorts. Additionally, some of their estimates for the wage rate are negative with the inclusion of children, which is nonsensical since Frisch coefficients capture pure substitution effects. This suggests a problem with the specification.

Another serious econometric violation is that the labor supply and commodity demand functions are not estimated simultaneously. As previously established by Abbot & Ashenfelter (1976) we cannot assume separability of these two functions. Not only do they implicitly assume separability in several of their regressions, they only use basic OLS in the estimation. When estimating a set of mutually dependant regressions the use of OLS will lead to inconsistent estimates. (Gujarati, 2003) It is therefore necessary to account for this not only in the specification, but in the procedure with which the model is estimated. In other words, it is econometrically appropriate to use a simultaneous estimation technique like seemingly unrelated regressions, three-stage least squares, or full information maximum likelihood estimation. Finally, because their specification is estimated separately they do not address the need for an appropriate identification and the possibility of simultaneity bias.

CHAPTER 3: INTERTEMPORAL ELASTICITY OF SUBSTITUTION

At first glance it may appear that the primary focus of this paper is to estimate a labor supply and consumption expenditure model. Although the estimation of this model plays a pivotal role, it is not the main objective. Ultimately, we seek estimates of the intertemporal elasticity of substitution for both labor [IES_L] and consumption [IES_C]. The goal of this chapter is to familiarize the reader with each of these parameters, making inference of their potential usefulness much easier. There are two conceptual parts that make up the following three sections, the first being an in depth discussion of what the *IES* is and the second being an overview of its significance.

3.1. IES_C

The IES_C is a parameter related to dynamic utility functions that essentially governs the curvature of indifference curves. In other words, it determines how the optimal consumption ratio $\left[\frac{c_{t+1}}{c_t}\right]$ varies with the marginal rate of substitution for consumption. Higher values of the IES_C imply that households become more willing to substitute away from saving and future consumption for current consumption in response to interest rate or price innovations. To see this, begin by defining a constant elasticity of substitution [CES] type utility function, *U*, and the IES_C as:

$$\mathrm{U} \equiv \frac{\mathrm{C}^{1-\sigma}}{1-\sigma}; \sigma > 0 \tag{3.1}$$

$$IES_C \equiv \left[\frac{\partial(\mathrm{Slope})\frac{\mathrm{C_{t+1}}}{\mathrm{C_t}}}{\partial\left(\frac{\mathrm{C_{t+1}}}{\mathrm{C_t}}\right)(\mathrm{Slope})}\right]^{-1} \equiv -\frac{\mathrm{U_C}}{\mathrm{C}\cdot\mathrm{U_{CC}}} \tag{3.2}$$

Where *Slope* is the slope of the indifference curve, U_C is the marginal utility of consumption $\left[\frac{\partial U}{\partial C}\right]$, and U_{CC} represents the change in marginal utility with respect to a change in consumption $\left[\frac{\partial^2 U}{\partial C^2}\right]$. C represents consumption, σ is a time invariant parameter that must be greater than 0, and t subscripts represent time periods. Additionally, differentiation of equation (3.1) gives $\frac{\partial U}{\partial C}$ as $C^{-\sigma}$and $\frac{\partial^2 U}{\partial C^2}$ as $-\sigma C^{-\sigma-1}$. Thus we calculate the IES_C, as per equation (3.2), to equal $\left[\frac{1}{\sigma}\right]$.

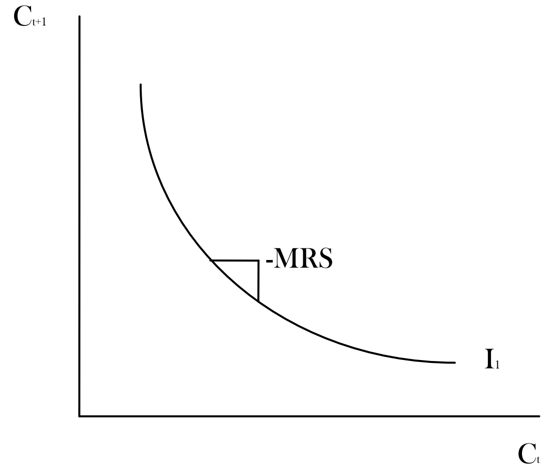
Now, to see how the IES_C affects the allocation of consumption in a dynamic framework, define the marginal rate of substitution for equation (3.1) as:

$$\mathrm{MRS} \equiv \frac{\mathrm{U_{C_t}}}{\mathrm{U_{C_{t+1}}}} = \frac{C_t^{-\sigma}}{C_{t+1}^{-\sigma}} = \left[\frac{\mathrm{C_{t+1}}}{\mathrm{C_t}}\right]^{\sigma} \tag{3.3}$$

Note that we ignore the rate of time preference here for simplicity. A graphical representation of (3.3) can be found on the following page.

Figure 3.1 shows the MRS in a two period setting as the negative of the slope of the indifference curve between period *t* and *t*+1. More intuitively, the marginal rate of substitution represents the subjective value of consumption at time *t* in terms of consumption at time *t*+1. It tells us how much consumption in the future an individual is willing to trade for consumption today. We find that for higher values of the MRS the indifference curve is steeper and households are willing to trade away more future

Figure 3.1



consumption for an additional unit of current consumption in response to interest rate and/or price innovations.

Now that we have established how the IES_C affects savings and consumption decisions, it is important to show *why* the IES_C affects savings and consumption decisions. The reason is that the IES_C determines the degree to which the utility function is concave. To see how this is so, start with equation (3.4). Note that the concavity or convexity of any function is determined by the combination of that function's first and second derivative signs. The standard non-satiation assumption and a negative U_{cc} imply the original function [*U*] must be concave.

$$U_{cc} = -\sigma C_i^{-\sigma-1} \tag{3.4}$$

Equation (3.4) also tells us that the second derivative depends on σ, which has already been shown as the key component of the IES_C. Therefore, the interaction of consumption and the level of σ affect how individuals value current consumption relative to future consumption.

People will thus alter savings decisions as their responsiveness to changes in the return on investment varies.

3.2. IES_L

The intertemporal elasticity of substitution for labor is interpreted similarly to the IES_C. In fact it would be possible, for the most part, to reproduce section 3.1 (with some minor changes in notation of course) within this section. For brevity's sake, however, we will not do that here and will instead highlight some key similarities and differences that exist between the two.

In demand economics utility is typically defined as a function of consumable goods like food and clothing, to more generally specified goods like 'X' and 'Y', to consumption itself. It is not, however, defined as a function of labor in any usual circumstance because labor itself does not provide utility. Based on the analysis in the previous section, how then do we define the IES_L?

A simple approach begins with equation (3.5):

$$\ell = T - L \tag{3.5}$$

Where T represents the individual's total time endowment in hours, L represents the amount of labor supplied in hours, and ℓ is the amount of 'leisure'. It is worth noting here that a more appropriate interpretation of ℓ is 'time away from work', and we shall refer to it as such from this point on. Although we do not typically think of ℓ as a good or service that individuals are able to consume like other commodities, in the demand literature it is often considered to be one. That said, it then becomes possible to apply the analysis in section 3.1 to the IES_L via the relationship that exists between 'time away from work' and the amount of labor supplied by an individual found in equation (3.5).

The IES_L then, is also a parameter related to dynamic utility functions responsive to changes in price[3]. However, as should be readily evident, the IES_L governs instead an individual's willingness to allocate *labor supply* over time. Thus, higher values of the IES_L imply that an individual's labor supply is more responsive to changes in the wage rate over time. In other words, evolutionary wage changes[4] yield greater tradeoffs between the hours of labor an individual supplies and their consumption of 'time away from work'.

3.3. In Sum

At this point it has been clearly stated and shown on several occasions that the *IES* is all about responsiveness, but it is natural to question the significance of this. Why does this 'responsiveness' warrant the time and effort involved in researching the methodology and empirical estimation of the *IES*? Well, the degree to which consumption and labor supply are responsive is of great interest to policy-makers because at the macro level there are incredible implications. According to the Bureau of Economic Analysis the 2006 U.S. gross domestic product was $13.246 trillion with nearly 70% of that comprised of domestic personal consumption expenditures. The significance becomes clear when, for example, the IES_C for total personal consumption expenditure equals something like .04. A very

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[3] The price of labor is of course represented by the wage rate.
[4] See Figure 5.1 in chapter five.

conservative 1% increase in prices then yields a .04% reduction in total personal consumption expenditure, the equivalent of a $3.7 billion drop in GDP. It does not stop there though. According to the IRS, federal revenue from individual income taxes for 2006 was $1.236 trillion. Although the progressive tax structure of the federal income tax and the fact that the response to a wage change is in hours of labor supplied prevent us from performing a quick calculation, it is clear that the affect of a wage change on labor supply is of a large magnitude.

CHAPTER 4: DATA

Before moving on to the modeling portion of this thesis the provision of details regarding data collected and used in the estimation is a measure of prudence we will take in this chapter. These details are particularly relevant here as they affect the interpretation and applicability of the model itself. We will start with a general discussion of the source, then proceed to the variables used in both the labor supply and commodity demand functions, and finalize the chapter with an account of the imputation procedure used to obtain estimates of total nondurable personal expenditure.

4.1. Panel Study of Income Dynamics

This particular study was undertaken by the University of Michigan in 1968 in an attempt to better understand poverty and its effects on the family. It consists of longitudinal data initially collected annually on 4,800 families, which eventually expanded to include more than 7,000 families in 2001. In terms of individuals, 65,000 people have been studied over as many as 36 years of their lives. The original PSID dataset included two independently drawn samples, one nationally representative and the other of low-income families. The nationally representative sample consisted of approximately 3000 households drawn randomly with equal probability from each of the 48 contiguous states. Another additional 2000 Latino households of Mexican, Puerto Rican, and Cuban descent were eventually included to better capture changes in immigration patterns. This would not be the last time that alterations would be made to the PSID.

4.2. Variables of Interest

The model to be developed in the next chapter is a system of equations for labor supply and commodity demand. Variables typically included in such models include hours of labor supplied, the wage rate, total expenditure and some measure of prices. This thesis, however, differs from the norm in that the two equations are estimated simultaneously with the implicit assumption that labor and consumption are not separable. Because of this assumption each equation requires a term linking one to the other. Details of this term will be developed later.

Within the PSID, the measure of labor supply is the individual's average hours worked per week during the previous year. For the wage rate, the head of household's average hourly earnings are calculated by dividing the labor income for the previous year by the total hours supplied in the previous year. The Consumer Price Index serves as a proxy for prices, and is supplied by the Bureau of Labor Statistics. The index is an annual US city average with 1982-1984 as the base period, which in addition to acting as a proxy is used to deflate consumption. In some instances we include the total number of children in the estimation so that more comparisons of our results can be made with those of BDI (1985). To discount both prices and wages we use 10 year T-bill rates from the Federal Reserve with

1981 as the base year. Individual fixed effects are captured with the use of binaries, while time effects are captured with a time trend. Finally, when appropriate, we instrument the natural log of discounted wages with age, age^2, and one period lagged prices and wages.

4.3. Imputing Consumption

One of the major obstacles to this research project has been the acquisition of consumption data because the PSID and all other major longitudinal data sets lack this information. Fortunately a solution to the consumption issue presented itself rather quickly after a brief search online. Blundell et al. (2006), make the best of the data available by devising an interesting technique specifically tailored, coincidentally, to the PSID. The imputation procedure involves the estimation of a food demand function using data from the Consumer Expenditure survey (CEX), inverting the demand equation, and solving for total non-durable consumption using data from the PSID.

This is a key differentiating factor between the estimation procedure in this thesis and that of BDI (1985) on which the model developed in the next chapter is based. Their solution to the absent consumption data was to not actually examine genuine households and their respective dynamics but to create synthetic cohorts from the British Family Expenditure Survey, which of course is not a panel. This is not ideal since "individual heterogeneity is basically summarized by cohort heterogeneity, which may be restrictive." (Blundell, Pistaferri, & Preston, 2006, p. 2)

The equation that serves as the foundation for the imputation procedure is a standard demand function for food. Blundell et al. (1985) write it as follows:

$$\tau(f_{i,x}) = D_{i,x}^{'}\beta + \gamma\eta(c_{i,x}) + e_{i,x} \tag{4.1}$$

Equation (4.1) says that the food expenditure ($f_{i,x}$) function (τ) is comprised of D (a vector of prices and conditioning variables), some function (η) of total non-durable expenditure ($c_{i,x}$), and $e_{i,x}$ which captures unobserved heterogeneity of food demand. Total non-durable expenditure is defined as "food (at home and away from home), alcoholic beverages and tobacco, services, heating fuel, transports (including gasoline), personal care, clothing and footwear, and rents." (Blundell, Pistaferri, & Preston, 2006, p. 16) Note that subscripts i and x from (4.1) indicate data used is for the i'th individual from the CEX, respectively. Also note that of all the variables in Equation (4.1) the only data not included in the PSID is individual total non-durable expenditure, the notation for which is $c_{i,p}$.

Once the food demand equation has been estimated using data from the CEX, the coefficients can then be used along with figures from the PSID to obtain an imputed value of consumption corresponding to the PSID data. It is important then to understand that in order for the process to work both the CEX and PSID samples have to be randomly drawn from the same population.

To see how the consumption data is imputed, start by inverting equation (4.1) and solving for total non-durable consumption, yielding:

$$\hat{c}_{i,x} = \eta^{-1}\left(\frac{\tau(f_{i,x}) - D_{i,x}^{'}\hat{\beta}}{\hat{\gamma}}\right) \tag{4.2}$$

Equation (4.2) defines *fitted* total non-durable consumption for the CEX. Equation (4.3) defines *imputed* total non-durable consumption for the PSID. Note the difference in subscripts.

$$\hat{c}_{i,p} = \eta^{-1}\left(\frac{\tau(f_{i,p}) - D_{i,p}^{'}\hat{\beta}}{\hat{\gamma}}\right) \tag{4.3}$$

$\hat{\gamma} \neq 0$ is assumed to be true for (4.2) and (4.3).

One important consideration is that an endogeneity problem may exist if total non-durable consumption decisions are made jointly with food expenditure decisions. In other words, if total non-durable consumption as an explanatory variable is correlated with the error term in Equation (4.1) an endogeneity problem will exist thus leading to inconsistent estimators. This issue can be resolved through the use of valid instruments. Blundell et al. (2006) use the hourly wage rate of the husband and wife (both by cohort, year, and education) as instruments for total consumption expenditure. The 2SLS process employed is relatively straightforward in that total non-durable real consumption is regressed on both of the real wage rates to obtain fitted values for consumption. The fitted values are then used in the estimation of the food demand equation.

Although available in Appendix B, it is useful at this point to provide the specifics of vector D. Included in it are the log prices of food, alcohol & tobacco, fuel & utilities, and the price of transportation all proxied with the appropriate CPI. Additionally, it includes an interaction term for the log of total-nondurable expenditure and each of several other demographic variables including education, year, and the number of children. Age, a quadratic in age, and family size also enter as explanatory variables meant to capture heterogeneity in the demand for food.

4.4. Data Screening

There are clearly limits to what can be accomplished in any given amount of time, so it is necessary under certain circumstances to make compromises at the expense of thoroughness. Because of this we decided to screen the data in a manner similar to Blundell et al. (2006), ultimately allowing for the use of their coefficient estimates in Table V of their paper. Ideally we would have been able to estimate equation (4.1), thereby allowing for more flexibility of the dataset. However, due to time restrictions it is not possible.

The screening process is the same for both the variables of interest within the model developed in chapter five and those variables used for the imputation of total non-durable consumption. It begins with the acquisition of data from the PSID's Data Center on each variable from 1968 to 1992. The first cut to the dataset is for interviews prior to 1979. After this, to follow suit with Blundell et al. (2006), we eliminate households with a female head and families with major changes in composition. These primarily include families where a divorce or death of a spouse occurs. Individuals without income data for more than 4 periods are eliminated in addition to those who receive top coding. We also eliminate families with a head of household younger than 30 or older than 65 in 1979. Finally, because the aforementioned SEO and Latino cohorts are not nationally representative they too are eliminated.

CHAPTER 5: THE MODEL

The material presented thus far has served to familiarize the reader with relevant literature in the field of labor supply and commodity demand, expand further upon a widely unknown parameter called the intertemporal elasticity of substitution, and lastly to detail from where and how the data we use is collected. In this chapter we begin with an overview of the little known Frisch demands, develop a tractable empirical model that can be used to obtain estimates of the *IES*, and end the chapter by discussing the contributions made here with our use of BDI's (1985) model.

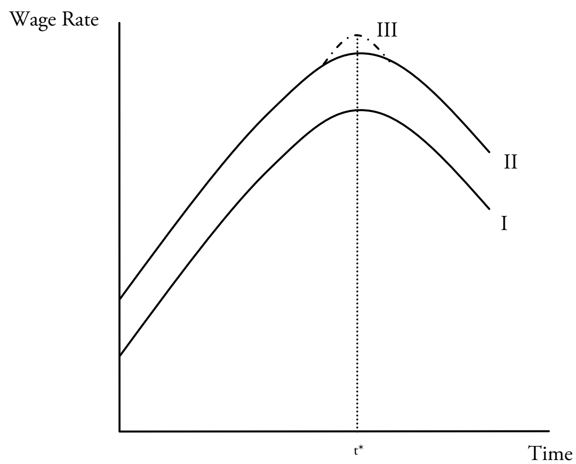
5.1. Why Frisch Demands?

Frisch demands have been mentioned several times throughout this text as having been employed by several economists including Heckman (1974), MaCurdy (1981), and BDI (1985). We identified the central differentiating factor among these and other functional forms of demand to be the holding constant of marginal utility of lifetime income. In this section of the chapter we explore Frisch demands more fully and how they apply to the life-cycle hypothesis.

Again, Frisch demands assume the marginal utility of lifetime income to be unobservable and constant allowing for not only the separation of anticipated and unanticipated wage changes but the treatment of differences among households as fixed effects. BDI (1985) recognize that neither compensated nor uncompensated labor supply functions are adequate when developing a tractable empirical specification. That it is in fact, as Heckman is the first to show, necessary to employ marginal utility of lifetime income constant labor supply and commodity demand functions.

Figure 5.1[5] tells a story of how wage changes affect three individuals' lifetime wage profiles and hence the labor supply of the individual. Starting with individual I, it is easy to

Figure 5.1



see that as time passes there is a natural progression of the wage rate. Due to such a progression, coupled with the assumption of 'perfect certainty', expected wage changes merely reflect a movement along the profile and are known as evolutionary wage changes. The significance of differentiating evolutionary wage changes from unexpected ones is that the former leave lifetime wealth and utility unchanged while the latter does not. Therefore, an evolutionary wage change leaves the marginal utility of lifetime income unaffected hence

[5] This is a reproduction of Figure 1 from MaCurdy (1981).

producing no income effect for the individual.

Unexpected wage changes affect an individual's wage profile and labor supply differently as seen in II and III's profiles. If at the beginning of the planning horizon I and II had the same profile, the graph shows that II experienced a permanent parametric wage change that shifts the lifetime wage and corresponding labor supply profile up. III experiences a parametric wage change at time t^*, but a windfall which affects the labor supply and wage profiles differently. The affect of the windfall can be seen as a 'blip' in the profile, but since it is of a transitory nature the wage change's effect on the profile is negligible and can be ignored. It is worth noting that the blip may affect the labor supply profile in a similar manner as the permanent parametric change but perhaps to a different degree.

To summarize, the framework of the life-cycle hypothesis necessitates the use of Frisch demands. Again it is because only these allow for the separation of anticipated and unanticipated wage changes as well as the treatment of individual differences as fixed effects. This is of course due to the fact that the marginal utility of lifetime income is held constant, thus removing income effects from the equation. Wage changes then, when employing Frisch demand functions, represent pure intertemporal substitution effects, are captured by the IES_L, and seen as movements along the labor supply profile.

5.2. The Model

A logical question one may ask is how to devise a model that employs Frisch labor supply and commodity demand functions. This is where the ingenuity of BDI (1985) becomes readily apparent as they demonstrate that the derivative of an individual profit function with respect to wage or price yields a Frisch labor supply or commodity demand function, respectively. A conceptually similar scenario is the derivation of an individual's minimum expenditure function with respect to prices, which of course yields a Hicksian demand.

In the context of this research these functions represent the profit an individual attains from selling utility to him or herself. But to better understand profit functions we first need to derive the price at which one sells utility. Begin by defining the standard maximization problem using (5.1) and (5.2):

$$V = \sum v_i(q_i) \qquad (i = 1, 2, \dots, n) \tag{5.1}$$

$$p * q = x \tag{5.2}$$

The subutilities given by $v_i(q_i)$ are monotone *increasing* in q_i as a result of the standard non-satiation property. V is the sum of the subutilities, i.e. total utility, and is maximized subject to the budget constraint in (5.2). The budget constraint defines expenditure on all goods, x, as the product of an (n x 1) vector of prices p and a (1 x n) vector of quantities q. The first order condition to this constrained maximization problem leads to the following definition of the marginal utility of q_i.

$$\frac{\partial v_i(q_i)}{\partial q_i} = \lambda p_i \tag{5.3}$$

Next, to find the price at which the individual sells utility to him or herself set λ equal to one over *r* as seen in (5.4). Intuitively lambda is the marginal utility of x, the reciprocal of which

[*r*] gives the change in total expenditure with respect to a one unit change in utility. In other words, *r* is the 'price' of utility.

$$\lambda p_i = \frac{p_i}{r} \tag{5.4}$$

Lastly, $\frac{\partial v_i(q_i)}{\partial q_i}$, defined as $v_i'(q_i)$, is invertible to give (5.5). This is because we apply the strict concavity assumption to the subutility functions defined previously as $v_i(q_i)$, which means that $v_i'(q_i)$ must be monotone *decreasing* in q_i. The fact that $v_i'(q_i)$ is monotone decreasing and hence invertible, simply allows for a unique solution for each q_i in terms of the price of utility, r.

$$q_i = f_i\left(\frac{p_i}{r}\right) \tag{5.5}$$

The price of utility is the key to understanding the profit function, and since the derivation of r has been identified it is useful at this point to make an intuitive comparison of the individual's profit function to that of a standard firm. The typical profit maximizing firm seeks to create as large a divide between the revenue it generates from sales and the cost of the goods sold. For the individual selling utility to his or herself it is no different since the goal is to maximize the difference between the 'revenue' of utility sales and the cost of utility production, with the cost of production here being total personal consumption expenditure. BDI specify the individual profit function in (5.6).

$$\pi(r, p) = \max\{rV - p * q; V = v(q)\} \tag{5.6}$$

The profit function above is homogenous of degree 1 in both the price of utility as well as commodity prices which follows from assuming the absence of money illusion. This assumption basically says that q is homogenous of degree 0 in commodity prices and lifetime income. But since r is related linearly to lifetime income, q is also homogenous of degree 0 in the price of utility. Therefore, since q is homogenous of degree 0 in r and p the profit function is homogenous of degree 1. Finally, the profit function requires the aforementioned set of subutility functions to be strictly concave.

By Sheppard's Lemma, differentiation of the profit function with respect to prices and wage rates is what finally yields the Frisch commodity demand and labor supply functions, respectively.

$$\frac{\partial \pi(r,p)}{\partial p_i} = -q_i \tag{5.7}$$

$$\frac{\partial \pi(r,p)}{\partial \mathrm{w}} = h \tag{5.8}$$

In the equations above q_i is the Frisch demand for good *i* while h represents the Frisch supply of labor in hours for a given household. Additionally, the Frisch labor supply functions implicitly assume labor force participation. If h is zero it implies that the value of time away from work is greater than the wage rate, which then results in zero labor supplied by the individual.

Note that the profit function is decreasing in the prices of commodities as evidenced by the negative sign on q_i. This makes intuitive sense as well since an increase in the price of a given commodity simultaneously results in an increase of the costs associated with utility production. The opposite is true of a change in the price of utility, r, ceteris paribus. If p_i remains constant, or increases less than proportionately to r, then an increase in the price of utility leads to higher 'revenue' and hence higher profits for the individual. The profit function is thus increasing in r.

Now that it has been shown how one generally arrives at Frisch labor supply and commodity demand functions it is worth mentioning a few properties of them. First, since the profit function is linear homogenous in p_i and r the Frisch labor supply and commodity demand functions are homogenous of degree zero. This means that there is no 'money illusion' since proportionate increases in p_i and r leave commodity demand and labor supply unchanged. The other property of interest is that of symmetry by way of Young's theorem. Therefore,

$$q_{ij} = q_{ji} \tag{5.9}$$

BDI continue on and present the framework of what has been discussed above under a lifecycle setting where, for the most part, it remains conceptually unchanged. The additional considerations for the dynamic version of the theoretical modeling include discounting where appropriate and how certainty, or the lack thereof, should be handled. The effect of uncertainty on the model begins with r.

The price of utility is the link between each period of an individual's life in terms of utility production and sales. By default r is the discounted value of the price of lifetime utility, with labor supply and consumption decisions made over the course of the lifecycle to ensure that it remains constant. r_t, however, is the period specific value of the price of utility and it follows a progressive evolution throughout the individual's life as in (5.10).

$$r_{t+1} = r_t(1 + i_t) \tag{5.10}$$

This, of course, is the certainty case where the individual knows the progression of the wage profile and hence has no need for future revisions of labor supply or consumption decisions.

If one decides to model labor supply and commodity demand using marginal utility of lifetime wealth constant functions, it is not difficult to incorporate uncertainty. Although the individual now attempts to maximize *expected* lifetime utility, the simplicity follows from the fact that it is only the progression of r_t we see change. Under uncertainty r_t follows a stochastic process given by 5.11.

$$E_t\left\{\frac{(1+i_t)r_t}{r_{t+1}}\right\} = 1 \qquad (5.11)$$

Although the reasoning behind the evolution of r varying between certainty and uncertainty may not be intuitively clear the important thing to remember is that if the individual has all relevant information at the beginning of the planning horizon, or if they have to take new information into consideration each period, as long as r remains constant the effect of wage changes on labor supply and commodity demand are estimated and interpreted similarly.

Before moving on it is worthwhile to make a few points about the price of utility. The first of which is that we expect to see r increase with younger individuals since on average people today face better economic conditions than in the past, and by better economic conditions the implication is a higher real wage rate at a similar age. Intuitively, it is reasonable to assume that if the individual has a higher real wage rate their lifetime wealth, real consumption, and price they are willing to pay for a unit of utility should also be higher. By equations (5.3) and (5.4), if utility increases strict concavity of the utility function and the decreasing monotonicity of $v_i'(q_i)$ require that marginal utility of q_i go down and r up, ceteris paribus. Lastly, inherited assets tend to produce a similar effect as well.

The next logical step is to derive an estimable empirical model based on the construct developed previously, but first it is necessary to deal with the fact that the price of utility is unobservable. Often times there are variables in which theory mandates inclusion in the preliminary theoretical modeling, but cannot be included in the estimation because there is no way of collecting data on that particular item. This is the case with r, and at first glance it may seem like a serious problem but in actuality the issue can be dealt with quite simply. All we require is that the variable can be treated as a fixed effect, which simply means that since it is constant for a given individual it can be first differenced out of the empirical model.

In order to maintain a steady flow throughout this chapter we forgo a complete derivation of the empirical model presented in (5.12a) – (5.13b). Instead, the interested reader will find it is presented adequately in BDI (1985). (5.12a/b) - (5.13a/b) represent the model under certainty and uncertainty, respectively. Note that we have modified these equations to reflect the fact that we use a true panel, and that no distinction between young and older children is made in our estimation.

$$h_t^i = \alpha_1^0 + \beta_1 \, ln \, \widetilde{w}_t^i + \theta_1 \sqrt{\frac{p_t}{w_t^i}} - \beta_1 \, ln \, \tilde{r}^i + \gamma_1 a_t^i + u_{1t}^c \quad (5.12a)$$

$$q_t^i = \alpha_2^0 + \beta_2 \, ln \, \tilde{p}_t + \theta_2 \sqrt{\frac{w_t^i}{p_t}} - \beta_2 \, ln \, \tilde{r}^i + \gamma_2 a_t^i + u_{2t}^c \quad (5.12b)$$

$$\Delta h_t^i = \Delta\alpha_{1t} + \beta_1 \, \Delta ln \, \widetilde{w}_t^i + \theta_1 \Delta \sqrt{\frac{p_t}{w_t^i}} + \gamma_1 \Delta a_t^i - \beta_1 \eta_t \quad (5.13a)$$

$$\Delta q_t^i = \Delta\alpha_{2t} + \beta_2 \, \Delta ln \, \tilde{p}_t + \theta_2 \Delta \sqrt{\frac{w_t^i}{p_t}} + \gamma_2 \Delta a_t^i - \beta_2 \eta_t \quad (5.13b)$$

Equation (5.12a) gives the average hours of labor supplied [h_t^i] by a particular individual as a function of the natural log of discounted wages, the price-wage ratio (the inclusion of which marks the assumption of non-separability of labor supply and commodity demand), the price of utility, and the number of young and old children. The individual specific average 'price of utility' [$\tilde{r}^i$] becomes part of the constant and is captured by the fixed effect estimators. (5.12b) gives the individual specific total real non-durable personal expenditure as a function of prices, the wage-price ratio, the price of utility, and the number of young and old children. Here, $\tilde{r}^i$ also becomes part of the constant and is captured by the fixed effects estimators. Remember, $\tilde{r}^i$ is constant under certainty since labor supply and consumption decisions are made at the beginning of the life-cycle to maximize total lifetime utility. Finally, the uncertainty case is given by (5.13a) and (5.13b), which are the first differenced forms of (5.12a) and (5.12b). Due to the existence of uncertainty there is a random component to the price of utility that becomes part of the error after first differencing, while this process removes the constant component as well.[6]

5.3. What Is The Difference?

It has been clearly stated that the model we use is essentially the same as that employed by BDI (1985). One may be left wondering what the contribution of this research is, or if it is merely a reproduction of their work. Well, there are two significant contributions to be made econometrically in the data that is used and the method with which the regressions are run. Firstly, the data set used is as previously stated a true panel

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[6] See BDI (1985) for details.

instead of synthetic cohorts, which obscures individual effects. This again allows for accurate individual heterogeneity to be captured. Secondly, unlike BDI (1985), both the certainty and uncertainty case use a simultaneous estimation method since when estimating a set of mutually dependant regressions the use of OLS individually will lead to inconsistent estimates. (Gujarati, 2003) The simultaneous technique we have chosen to employ under both certainty and uncertainty is full information maximum likelihood since it is provides the flexibility to perform instrumentation with the presence of endogeneity. More details of the estimation are of course provided in chapter 6 with the results of the empirical estimation.

CHAPTER 6: EMPIRICAL RESULTS

At this point we have thoroughly laid out a theoretical framework and empirical model that are admittedly both complex and highly detailed. We now move on to the culmination of our work with the results of the estimation process. As earlier chapters have, this one follows a logical progression with the estimated regressions presented first, after which we conclude this thesis with a section that provides a brief overarching synopsis of everything that has been discussed, from key aspects of the lifecycle hypothesis to the empirical results of this chapter.

6.1. Estimation

We begin with Table 6.1 which lists the basic OLS results for the labor supply and commodity demand regressions. Aside from prices which we have proxied with the CPI, the estimates in this table and those that follow are calculated using weekly values of each variable. Note that check marks indicate whether or not dummies are included to capture individual fixed effects (c), cyclical effects (y), or both. Finally, the significance of separability of labor supply and commodity demand has been discussed already. To avoid this erroneous assumption a term must be included in the model that implicitly links the two equations; this term is the price-wage ratio mentioned in the last chapter. We exclude it

Table 6.1

Certainty

Labor Supply Regressions: Levels (Separable)

OLS Parameter Estimates							
Eq.	Constant	*ln* $\widetilde{w}$	*Children*	c	y	R^2	d.w.
6.101	7.69	5.81* (.394)	-	-	-	.113	1.98
6.102	-27.24	10.22* (.528)	-	√	-	.667	1.97
6.103	6.72	5.88* (.397)	-	-	√	.117	1.99
6.104	-30.67	10.61* (.530)	-	√	√	.675	2.01
6.105	6.87	5.78* (.391)	1.28* (.258)	-	-	.123	2.01
6.106	-28.52	10.41* (.529)	1.679* (.475)	√	-	.670	1.98
6.107	6.55	5.82* (.394)	1.23* (.262)	-	√	.128	2.02
6.108	-30.78	10.65* (.531)	.856** (.511)	√	√	.676	2.01

Personal Consumption Expenditure Regressions: Levels (Separable)

OLS Parameter Estimates							
Eq.	Constant	*ln* $\tilde{p}$	*Children*	c	y	R^2	d.w.
6.109	103.42	-310.88* (13.53)	-	-	-	.185	1.34
6.110	97.28	-363.50* (13.34)	-	√	-	.488	1.50
6.111	86.01	-328.99* (13.68)	15.29* (2.28)	-	-	.201	1.35
6.112	102.27	-356.32* (14.36)	-6.95 (5.14)	√	-	.489	1.51

* and ** indicate statistical significance at the 95% and 90% confidence levels, respectively. a and b, when present, identify corresponding equations. Check marks indicate the inclusion of individual fixed effects or cyclical dummies. Standard errors are in parenthesis beneath the respective coefficient.

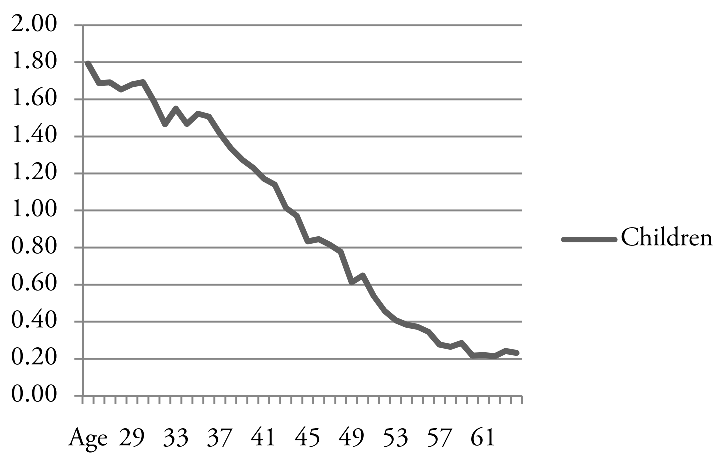
here, assuming separability, so that parallels can be more easily drawn among our results and those of BDI (1985).

Starting with the initial labor supply regressions from BDI (1985), their results show the relationship between hours supplied and the cyclical dummy is statistically significant. This implies, of course, that there are cyclical effects on the hours and wages of workers. However, contrary to the results provided by BDI (1985) where they state explicitly that the role of children and cyclical effects leave no role for the discounted wage rate in the determination of hours, the effect of time in 6.103 and 104 and children and time in 6.107 and 108 on the wage rate's explanatory power is marginal. In fact, children barely test significant at a reasonable level with the inclusion of the time and individual fixed effect dummies. The same holds true for the effect of the individual fixed effects dummies on the wage coefficient, however most of these did not test significant at any standard level.

BDI (1985) suggest that the results of the commodity demand function are not particularly interesting because prices do not follow a pronounced life-cycle pattern. There are some important implications, however, that can be derived from the results. We find that the coefficients on prices are consistently negative and significant to a very high degree. Additionally, the explanatory power of children is eliminated with the inclusion of individual fixed effects dummies. This in fact suggests that children may not be very good determinant of life-cycle consumption as BDI (1985) suggest, and that individual non-price factors other than the number of children influence it. This is also evidenced by Figure 6.1 which gives the average number of children by age.

The next round of estimation in Table 6.2 involves the uncertainty case where we re-estimate the specifications from Table 6.1 in differenced form. We first difference the regression in the uncertainty case because unlike under certainty there is now a random component to the price of utility. The random component alters the 'evolution' of r[7] and so again it must be dealt with by differencing, which in turn removes the fixed component and forces the random one to become part of the error term. Because r is not constant it can no longer be assumed that the fixed effects estimators capture the price of utility. Note that endogeneity is introduced in the uncertainty case and as such we follow suit with BDI (1985) by instrumenting the first differenced natural log of discounted wages with age, age^2, and both one period lagged wages and prices.

Figure 6.1



The BDI (1985) results are again not supportive of the life-cycle hypothesis with older children and the year dummies [cyclical effects] leaving not much of a role for the discounted wage rate. Once more our results contradict theirs for the labor supply regression with the fact that it is *only* the wage rate that tests significant in equations from 6.201-6.204 and 6.207-6.211. Additionally, they find that year to year changes in the wage rate are

[7] See BDI (1985) for more on this.

negatively correlated with hours, which contrasts with the positive values of their initial estimation. Ours are of course still positive and once again provide more evidence in favor of

Table 6.2

Uncertainty

Labor Supply & Personal Consumption Expenditure Regressions: First-Differences (Separable)

OLS Parameter Estimates

Eq.	Constant	*Δln* $\tilde{w}$	*ΔChildren*	y	R^2	d.w.
6.201	-.763	11.00* (.573)	-	-	.225	2.02
6.202	.783	10.97* (.576)	-	√	.232	2.04
6.203	-.805	11.01* (.574)	-.496 (.819)	-	.225	2.03
6.204	.758	10.97* (.576)	-.619 (.822)	√	.232	2.04

OLS Parameter Estimates

Eq.	Constant	*Δln* $\tilde{p}$	*ΔChildren*	y	R^2	d.w.
6.205	65.03	519.31* (117.95)	-	-	.010	1.70
6.206	65.04	525.69* (118.33)	-6.01 (8.76)	-	.010	1.70

2SLS Parameter Estimates

Eq.	Constant	*Δln* $\tilde{w}$	*ΔChildren*	y	R^2	d.w.
6.207	-.936	16.66* (7.44)	-	-	.004	1.91
6.208	1.06	11.29 (8.23)	-	√	.014	1.95
6.209	-.963	16.74* (7.46)	-.265 (.851)	-	.004	1.91
6.210	1.03	11.45 (8.24)	-.381 (.817)	√	.014	1.95

* and ** indicate statistical significance at the 95% and 90% confidence levels, respectively. a and b, when present, identify corresponding equations. Check marks indicate the inclusion of cyclical dummies. Standard errors are in parenthesis beneath the respective coefficient.

the theory.

We now reintroduce the restriction that labor supply and commodity demand are non-separable with the implication that there are intratemporal substitution effects. Tables 6.3 and 6.4 provide many permutations under the assumption of certainty (note the inclusion of the price-wage ratio). Furthermore, as previously mentioned one of the methods we use in an attempt to improve upon the approach taken by BDI (1985) is simultaneous equation estimation. The reasoning behind this, as already explained in the model section, is that it is econometrically appropriate to do so in order to arrive at consistent estimates. We therefore provide not only the estimates using OLS but seemingly full information maximum likelihood [FIML] simultaneous equation estimation. There are of course multiple techniques that are up to the task at hand, however we choose to estimate the regressions under certainty using FIML because it is ideal if one wishes "to preserve the spirit of simultaneous-equation models…" (Gujarati, 2003, p. 763)

Beginning with the OLS results for the labor supply regression, some of the BDI (1985) estimates once again are not particularly supportive of the theory. There is a negative coefficient on the wage rate and hence the IES_L, while the term that 'connects' labor supply and commodity demand is not significant. The rest of their estimates are more supportive once the time dummies enter the regression in that the wage rate coefficients are positive and significant. There is, however, a conflict with Heckman (1974) and the lifecycle hypothesis because the price-wage ratio is positive, which suggests a complementary relationship between leisure and commodities. The reasoning behind this is that as prices increase relative to the wage rate we see an increase in labor and therefore a reduction of leisure. If leisure declines and we assume a negative relationship between prices and commodities, there

Table 6.3

Certainty

Labor Supply Regressions: Levels (Non-Separable)

		OLS Parameter Estimates						
Eq.	Constant	$ln\ \tilde{w}$	$\sqrt{p/w}$	*Children*	c	y	R^2	d.w.
6.301	3.14	6.40* (.803)	17.70 (20.85)	-	-	-	.114	1.98
6.302	-49.62	13.17* (.875)	72.52* (17.19)	-	√	-	.671	1.96
6.303	-3.32	7.15* (.867)	37.08** (22.47)	-	-	√	.119	2.00
6.304	-73.11	16.13* (.941)	129.78* (18.43)	-	√	√	.687	2.03
6.305	.233	6.64* (.799)	25.74 (20.76)	1.31* (.259)	-	-	.127	2.01
6.306	-55.81	14.02* (.889)	87.30* (17.39)	2.14* (.479)	√	-	.676	1.97
6.307	-3.90	7.14* (.862)	38.61** (22.33)	1.23* (.262)	-	√	.130	2.02
6.308	-73.82	16.26* (.942)	131.51* (18.43)	1.02* (.502)	√	√	.688	2.03

Personal Consumption Expenditure Regressions: Levels (Non-Separable)

		OLS Parameter Estimates						
Eq.	Constant	$ln\ \tilde{p}$	$\sqrt{w/p}$	*Children*	c	y	R^2	d.w.
6.309	103.51	-289.77* (14.91)	.245 (.373)	-	-	-	.183	1.38
6.310	174.27	-363.32* (17.29)	-1.77* (.660)	-	√	-	.527	1.54
6.311	87.73	-305.57* (14.96)	.200 (.369)	14.74* (2.38)	-	-	.201	1.40
6.312	176.89	-359.21* (18.75)	-1.79* (.662)	-3.32 (5.85)	√	-	.527	1.54

* and ** indicate statistical significance at the 95% and 90% confidence levels, respectively. a and b, when present, identify corresponding equations. Check marks indicate the inclusion of individual fixed effects or cyclical dummies. Standard errors are in parenthesis beneath the respective coefficient.

is a corresponding drop in the consumption of both goods.

At any rate, our results in 6.3 are similar in some ways and different in others from BDI (1985) in that wages and the price-wage ratio are positive and significant in most cases with only a few exceptions. This of course yields a similar conflict with Heckman's (1974) findings. Although the children repeatedly test significant their influence, as well as that of the individual fixed effects and time dummies, on the significance of the wage rate is non-existent.

The consumption side of the picture is mixed as well. Although prices test negative and highly significant in each of the regressions we obtain both a negative and positive sign on the wage-price ratio. The negative sign suggests a substitute relationship exists between leisure and commodities while the positive sign suggests they are compliments, thus creating a conflict not only within the commodity demand equation but with the labor supply equation as well. Finally, children again play a minor role as they are only significant with the inclusion of the time dummy.

Moving on to Table 6.4, we find that the analysis of the FIML estimates is quite similar in that we do not have any sign or significance issues with the wage rate. Again prices test significant consistently (here in every regression) without any sign issues as BDI (1985) found. The wage-price and price-wage ratios continue to have similar sign discrepancies, though test significant in most cases.

Finally in Tables 6.5 and 6.6 we have the results for the uncertainty case, with the 2SLS and OLS estimates in the first table and the FIML estimates in the latter. The FIML

Table 6.4

Certainty

Labor Supply Regressions: Levels (Non-Separable)

FIML Parameter Estimates							
Eq.	Constant	$ln\ \tilde{w}$	$\sqrt{p/w}$	*Children*	c	y	d.w.
6.401a	2.83	6.45* (.800)	18.57 (20.71)	-	-	-	1.97
6.402a	-48.32	12.98* (.773)	71.43* (15.14)	-	√	-	1.97
6.403a	-3.74	7.19* (.861)	37.60** (22.28)	-	-	√	1.98
6.404a	-71.96	15.96* (.829)	128.92* (16.18)	-	√	√	2.05
6.405a	-.278	6.71* (.795)	27.04 (20.60)	1.40* (.259)	-	-	2.00
6.406a	-54.72	13.85* (.785)	86.67* (15.30)	2.21* (.426)	√	-	1.98
6.407a	-4.27	7.19* (.855)	39.16** (22.13)	1.33* (.262)	-	√	2.00
6.408a	-72.69	16.09* (.830)	130.71* (16.17)	1.07* (.446)	√	√	2.05

Personal Consumption Expenditure Regressions: Levels (Non-Separable)

FIML Parameter Estimates							
Eq.	Constant	$ln\ \tilde{p}$	$\sqrt{w/p}$	*Children*	c	y	d.w.
6.401b	103.34	-291.09* (14.89)	.231 (.372)	-	-	-	1.38
6.402b	174.25	-364.54* (15.28)	-1.79* (.584)	-	√	-	1.54
6.403b	103.65	-289.76* (14.90)	.238 (.372)	-	-	-	1.38
6.404b	174.38	-363.32* (15.28)	-1.77* (.584)	-	√	-	1.54
6.405b	87.69	-306.06* (14.94)	.193 (.368)	14.75* (2.37)	-	-	1.40
6.406b	176.82	-360.48* (16.57)	-1.81* (.585)	-3.18* (5.17)	√	-	1.54
6.407b	87.81	-305.56* (14.95)	.196 (.368)	14.74* (2.37)	-	-	1.40
6.408b	177.03	-359.2* (16.57)	-1.80* (.585)	-3.32 (5.17)	√	-	1.54

* and ** indicate statistical significance at the 95% and 90% confidence levels, respectively. a and b, when present, identify corresponding equations. Check marks indicate the inclusion of individual fixed effects or time dummies. Standard errors are in parenthesis beneath the respective coefficient.

estimation here differs procedurally from the certainty case because of the instrumentation stage included. We simply identify the endogenous variable and the instruments as is done in the 2SLS estimation. These figures in Table 6.5 are not as strong as the previous results. We find that the wage rate is significant at the 90% confidence level at best, while the

Table 6.5

Uncertainty

Labor Supply Regressions: First-Differences (Non-Separable)

2SLS Parameter Estimates

Eq.	Constant	$\Delta ln\ \tilde{w}$	$\Delta\sqrt{p/w}$	*ΔChildren*	y	R^2	d.w.
6.501	-.773	13.36** (7.91)	-93.31* (11.07)	-	-	.061	1.90
6.502	1.02	8.36 (8.71)	-93.30* (10.54)	-	√	.074	1.95
6.503	-.839	13.54** (7.94)	-93.72* (11.12)	-.652 (.906)	-	.061	1.91
6.504	.953	8.66 (8.73)	-93.70* (10.57)	-.727 (.865)	√	.074	1.95

Personal Consumption Expenditure Regressions: Levels (Non-Separable)

OLS Parameter Estimates

Eq.	Constant	$\Delta ln\ \tilde{p}$	$\Delta\sqrt{w/p}$	*ΔChildren*	y	R^2	d.w.
6.505	58.61	418.57* (154.79)	-2.43* (.741)	-	-	.014	1.72
6.506	58.61	424.31* (155.47)	-2.44* (.741)	-4.19 (10.23)	-	.015	1.72

* and ** indicate statistical significance at the 95% and 90% confidence levels, respectively. a and b, when present, identify corresponding equations. Check marks indicate the inclusion of cyclical dummies. Standard errors are in parenthesis beneath the respective coefficient.

inclusion of the time dummies eliminates the significance of the wage rate altogether. As suggested by BDI (1985), this indicates that it is likely cyclical effects play a more important role in the determination of hours. We do however have consistency with the negative sign and significance of the cross-price effects in both the commodity demand and labor supply equations in Table 6.5. An important point briefly touched upon earlier regarding negative cross-price effects is that they are supportive of Heckman, Thurow's, and Nagatani's results which are supportive of the theory. Finally, the price estimates test significant every time,

Table 6.6

Uncertainty

Labor Supply Regressions: First-Differences (Non-Separable)

FIML Parameter Estimates

Eq.	Constant	$\Delta ln\ \tilde{w}$	$\Delta\sqrt{p/w}$	*ΔChildren*	y	d.w.
6.605a	-.871	15.94* (.915)	124.14* (15.62)	-	-	1.96
6.606a	.870	16.21* (.921)	129.38* (15.71)	-	√	1.98
6.607a	-.867	15.95* (.916)	124.20* (15.66)	.049 (.816)	-	1.96
6.608a	.864	16.20* (.921)	129.24* (15.74)	-.138 (.816)	√	1.98

Personal Consumption Expenditure Regressions: Levels (Non-Separable)

FIML Parameter Estimates

Eq.	Constant	$\Delta ln\ \tilde{p}$	$\Delta\sqrt{w/p}$	*ΔChildren*	y	d.w.
6.605b	72.61	603.32* (167.96)	-2.95* (.781)	-	-	1.71
6.606b	71.95	593.11* (168.00)	-2.95* (.781)	-	-	1.71
6.607b	72.61	603.00* (168.48)	-2.95* (.781)	.231 (10.60)	-	1.71
6.608b	71.95	592.75* (168.52)	-2.95* (.781)	.282 (10.60)	-	1.71

* and ** indicate statistical significance at the 95% and 90% confidence levels, respectively. a and b, when present, identify corresponding equations. Check marks indicate the inclusion of cyclical dummies. Standard errors are in parenthesis beneath the respective coefficient.

but the sign is incorrect.

Table 6.6 provides what we consider the most defensible results since earlier estimates may be biased and inconsistent. The reasoning behind this assessment is that uncertainty is a serious factor that should not be considered lightly in the design of a model. Thus, the empirical framework behind the uncertainty model is the most complete, and coupled with the econometrically appropriate simultaneous estimation we have a sound set of results. The results here are consistent with those of the earlier estimations in that the wage rate is consistently positive and significant. Children and the time dummies again play no significant role contradicting the results of BDI (1985). However, the labor supply and commodity demand functions again yield cross-price effects that are inconsistent with each other. Another snag in the results is that although prices consistently test significant, like BDI (1985) the sign has a counter intuitive positive sign yet again.

Finally, we present the estimates of the IES_C and the IES_L in Table 6.7. The results are organized so that the reader will find the respective equation the estimate is derived from in the leftmost column, the coefficient used in the middle, and the estimate in the rightmost column. Our results for the IES_L range from a low of about .16, which is along the same lines as BDI (1985) and MaCurdy (1981), to a relatively high .46. The latter value, although high, is along the same lines as BDI (1985) as well. Those for the IES_C are significantly higher than the results of BDI (1985) and range from -1.35 to 2.81. Note that the range is skewed because of the ‘incorrect’ sign on the price coefficient.

6.2. Conclusions

This thesis has provided an in depth analysis of the life-cycle hypothesis, which as stated previously was first introduced by Modigliani & Brumberg in 1954. It is a seminal

Table 6.7

IES Estimates

Eq.	ln w	IES_L	Eq.	ln p	IES_C	Eq.	Δln w	IES_L	Eq.	Δln p	IES_C
6.101	5.81	0.16	6.109	-310.88	-1.45	6.201	11.00	0.30	6.205	519.31	2.42
6.102	10.22	0.28	6.110	-363.50	-1.69	6.202	10.97	0.30	6.206	525.69	2.45
6.103	5.88	0.16	6.111	-328.99	-1.53	6.203	11.01	0.30	6.505	418.57	1.95
6.104	10.61	0.29	6.112	-356.32	-1.66	6.204	10.97	0.30	6.506	424.31	1.97
6.105	5.78	0.16	6.309	-289.77	-1.35	6.207	16.66	0.45	6.605b	603.32	2.81
6.106	10.41	0.28	6.310	-363.32	-1.69	6.208	11.29	0.31	6.606b	593.11	2.76
6.107	5.82	0.16	6.311	-305.57	-1.42	6.209	16.74	0.46	6.607b	603.00	2.81
6.108	10.65	0.29	6.312	-359.21	-1.67	6.210	11.45	0.31	6.608b	592.75	2.76
6.301	6.40	0.17	6.401b	-291.09	-1.35	6.501	13.36	0.36			
6.302	13.17	0.36	6.402b	-364.54	-1.70	6.502	8.36	0.23			
6.303	7.15	0.20	6.403b	-289.76	-1.35	6.503	13.54	0.37			
6.304	16.13	0.44	6.404b	-363.32	-1.69	6.504	8.66	0.24			
6.305	6.64	0.18	6.405b	-306.06	-1.42	6.605a	15.94	0.43			
6.306	14.02	0.38	6.406b	-360.48	-1.68	6.606a	16.21	0.44			
6.307	7.14	0.19	6.407b	-305.56	-1.42	6.607a	15.95	0.44			
6.308	16.26	0.44	6.408b	-359.20	-1.67	6.608a	16.20	0.44			
6.401a	6.45	0.18									
6.402a	12.98	0.35									
6.403a	7.19	0.20									
6.404a	15.96	0.44									
6.405a	6.71	0.18									
6.406a	13.85	0.38									
6.407a	7.19	0.20									
6.408a	16.09	0.44									

L and C subscripts denote the intertemporal elasticity of substitution estimate is for labor supply and consumption, respectively.

piece of work that has spurred countless works attempting to make sense of and explain long run labor supply and consumption decisions at the macroeconomic level. Conceptually the hypothesis is a simple assessment that individuals make decisions at the nascent of a planning horizon for their entire lifetime (i.e. they formulate the consumption and labor supply profiles representative of those in Figure 2.3 from the Literature Review) in such a way as to maximize total lifetime utility. There have been many attempts to model either the consumption side or the labor side of the issue, but as has been shown by Browning & Meghir (1991) they cannot be assumed separable. Others like Abbot & Ashenfelter (1976) have attempted to estimate both, but are subject to other violations like a static framework.

Not all is lost however, with the tractable empirical model of Browning, Deaton, and Irish (1985) we finally have a framework that incorporates all the necessary aspects of the hypothesis. Nonetheless, there are always ways to improve upon previous works. In the case of BDI, better data and estimation procedures could be employed. These are the contributions that this thesis has attempted to make with the use of the Panel Study of Income Dynamics and the FIML estimation. As it turns out these improvements have in some ways turned the tides in favor of the lifecycle hypothesis, at least when compared to the results of BDI (1985). The roles of children and business cycles and their effects on the wage rate and its explanatory power are largely diminished. Wages seem to be a much better determinant of labor supply than BDI (1985) give it credit for.

The results for the consumption side of the picture are not quite as strong due to the sign issues on the price coefficients, but as previously mentioned prices do not follow a pronounced lifecycle pattern. There are a few alternative views one can take of this information. It may be the case that one must simply include the consumption side to obtain consistent estimators for the labor supply regression. And that it does not much matter what the results for consumption are. However, we suspect that this explanation may not satisfy those who wish to understand consumption over the lifecycle. Another possible explanation for the lackluster demand results could be an issue with identification and simultaneity bias. Or maybe there was issue with the imputation procedure that the lack of

Figure 6.2

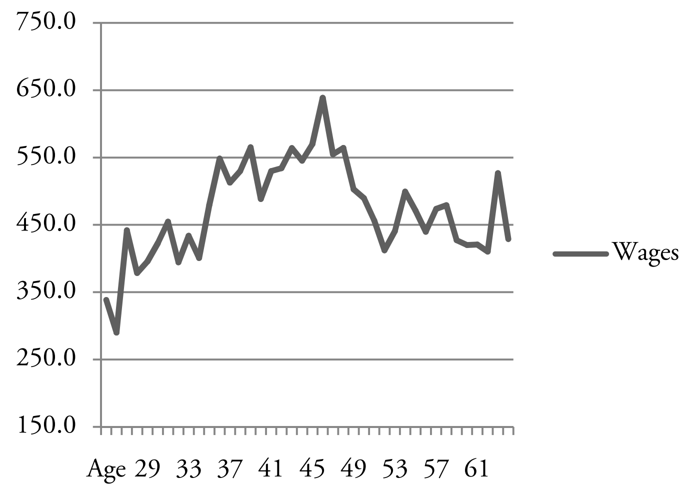


Figure 6.3

consumption data necessitated. Whatever the case, we have said before there are always ways to improve upon the works of others. The approach taken in this thesis could possibly be taken one step further by dealing with the identification issue or maybe with the re-estimation of the food expenditure function used by Blundell et al. (2006). The latter would allow the researcher to more specifically tailor the data set to the task at hand.

In any event, we have shown here that Modigliani & Brumberg's 53 year old work cannot be rejected by simply attributing labor supply to the number of offspring a family has, as suggested by BDI (1985). The data definitely shows a pronounced lifecycle pattern which can be seen in Figures 6.2 and 6.3, as well as the results from our estimation. Lastly, it is possible that business cycles play a significant role, but it is our opinion the results of this thesis have not shown much in favor of it.

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APPENDIX A: LITERATURE REVIEW DERIVATIONS

Equations 2.8 & 2.9

Given

$$G = G\left\{\sum_{t=0}^{T} \frac{1}{(1+\rho)^{t}} U\big[C(t), L(t)\big]\right\} \tag{A.1}$$

$$A(0) + \sum_{t=0}^{T} R(t)N(t)W(t) - \sum_{t=0}^{T} R(t)C(t) = 0 \tag{A.2}$$

<u>Step 1</u> – Set up the Lagrange. Note, additivity is assumed and thus *G* disappears.

$$\Psi = \sum_{t=0}^{T} \frac{1}{(1+\rho)^{t}} U\big[C(t), L(t)\big] + \lambda\left[A(0) + \sum_{t=0}^{T} R(t)N(t)W(t) - \sum_{t=0}^{T} R(t)C(t)\right] \tag{A.3}$$

<u>Step 2</u> – Differentiate with respect to the choice variables and set equal to zero.

$$\frac{\partial \Psi}{\partial C(t)} \Rightarrow \frac{1}{(1+\rho)^{t}} \frac{\partial U}{\partial C(t)} - \lambda R(t) = 0 \tag{A.4}$$

$$\frac{\partial \Psi}{\partial L(t)} \Rightarrow \frac{1}{(1+\rho)^{t}} \frac{\partial U}{\partial L(t)} + \lambda R(t)W(t) = 0 \tag{A.5}$$

<u>Step 3</u> – Solve for $\frac{\partial U}{\partial C(t)}$ and $\frac{\partial U}{\partial L(t)}$ given by U_1 and U_2, which yields 2.8 and 2.9.

$$U_1 = \lambda R(t)(1+\rho)^{t} \tag{A.6}$$

$$U_2 \geq \lambda W(t)R(t)(1+\rho)^{t} \tag{A.7}$$

Equation 2.12

Given

$$U_2[C(t), L(t)] = \lambda W(t)R(t)(1+\rho)^t \quad \text{A.8}$$

$$U_i[C_i(t), L_i(t)] = Y_{1i}(t)[C_i(t)]^{\omega_1} - Y_{2i}(t)[N_i(t)]^{\omega_2} \quad \text{A.9}$$

Step 1 – Differentiate equation A.9 with respect to *$L_i(t)$*, keeping in mind that *$N(t)=L^*-L(t)$*.

$$\frac{\partial U_i}{\partial L_i(t)} = \omega_2 Y_{2i}(t)[N_i(t)]^{\omega_2 - 1} \quad \text{A.10}$$

Step 2 – Set the right hand side from equation A.10 equal to the right hand side of equation A.8, take logs, and solve for *$lnN_i(t)$*.

$$\omega_2 Y_{2i}(t)[N_i(t)]^{\omega_2 - 1} = \lambda W_i(t)R(t)(1+\rho)^t \quad \text{A.11}$$

$$\ln\omega_2 + \ln Y_{2i}(t) + (\omega_2 - 1)\ln N_i(t) = \ln\lambda_i + \ln W_i(t) + \ln R(t) + \ln(1+\rho)^t \quad \text{A.12}$$

$$\ln N_i(t) = \frac{1}{(\omega_2 - 1)}\left[\ln\lambda_i + \ln W_i(t) + \ln R(t) + \ln(1+\rho)^t - \ln Y_{2i}(t) - \ln\omega_2\right] \quad \text{A.13}$$

Step 3 – Define δ as $\frac{1}{(\omega_2 - 1)}$, $\ln Y_{2i}(t)$ as $[\sigma_i - u_i^*(t)]$, F_i as $\delta(\ln\lambda_i - \sigma_i - \ln\omega_2)$, $u_i^*(t)$ as $\frac{u_i(t)}{\delta}$ and substitute where appropriate.

$$lnN_i(t) = F_i + \delta\sum_{k=0}^{t}[\rho - r(k)] + \delta lnW_i(t) + u_i(t) \quad \text{A.14}$$

Equation 2.14

Given

$$U_1[C(t), L(t)] = \lambda R(t)(1+\rho)^t \quad \text{A.15}$$

$$U_i[C_i(t), L_i(t)] = Y_{1i}(t)[C_i(t)]^{\omega_1} - Y_{2i}(t)[N_i(t)]^{\omega_2} \quad \text{A.16}$$

<u>Step 1</u> – Differentiate equation A.16 with respect to $C_i(t)$.

$$\frac{\partial U_i}{\partial C_i(t)} = \omega_1 Y_{1i}(t)[C_i(t)]^{\omega_1 - 1} \quad \text{A.17}$$

<u>Step 2</u> – Set the right hand side from equation A.17 equal to the right hand side of equation A.15, take logs, and solve for $lnN_i(t)$.

$$\omega_1 Y_{1i}(t) C_i(t)^{\omega_1 - 1} = \lambda_i R(t)(1+\rho)^t \quad \text{A.18}$$

$$ln\omega_1 + lnY_{1i}(t) + (\omega_1 - 1) lnC_i(t) = ln\lambda_i + lnR(t) + \ln(1+\rho)^t \quad \text{A.19}$$

$$lnC_i(t) = \frac{1}{(\omega_1 - 1)}[ln\lambda_i + lnR(t) + ln(1+\rho)^t - lnY_{1i}(t) - ln\omega_1] \quad \text{A.20}$$

<u>Step 3</u> – Define η as $\frac{1}{(\omega_1 - 1)}$, $lnY_{1i}(t)$ as $[\sigma_i - u_i^*(t)]$, V_i as $\eta(\ln\lambda_i - \sigma_i - \ln\omega_1)$ and substitute where appropriate.

$$lnC_i(t) = V_i + \eta \sum_{k=0}^{t} [\rho - r(k)] + u_i(t) \quad \text{A.22}$$

Equations 2.21 and 2.22

Given

$$\sum p_i x_i = wh + y \quad \text{A.23}$$

$$u = B_0 \ln(l - \gamma_i) + \sum B_i \ln(x_i - \gamma_i) \quad \text{A.24}$$

<u>Step 1</u> – Set up the Lagrange and differentiate with respect to x_i and l. Note that $dlnx = \frac{dx}{x}$.

$$\Psi = B_0 \ln(l - \gamma_i) + \sum B_i \ln(x_i - \gamma_i) - \lambda[\sum p_i x_i - wh - y] \quad \text{A.25}$$

$$\frac{\partial \Psi}{\partial x_i} \Longrightarrow B_i \frac{d(x_i - \gamma_i)}{d(x_i)} \frac{1}{(x_i - \gamma_i)} - \lambda p_i = 0$$ A.26

$$\frac{\partial \Psi}{\partial l} \Longrightarrow B_0 \frac{d(l - \gamma_l)}{d(l)} \frac{1}{(l - \gamma_l)} - \lambda w = 0$$ A.27

Step 2 – Solve for λp_i and λw.

$$\lambda p_i = B_i \frac{d(x_i - \gamma_i)}{d(x_i)} \frac{1}{(x_i - \gamma_i)} = B_i \frac{(1-0)}{(x_i - \gamma_i)} = \frac{B_i}{(x_i - \gamma_i)}$$ A.28

$$\lambda w = B_0 \frac{d(l - \gamma_l)}{d(l)} \frac{1}{(l - \gamma_l)} = B_0 \frac{(1-0)}{(l - \gamma_l)} = \frac{B_0}{(l - \gamma_l)}$$ A.29

Equation 2.23

Given

$$\lambda p_i = \frac{B_i}{(x_i - \gamma_i)}$$ A.30

$$\lambda w = \frac{B_0}{(l - \gamma_l)}$$ A.31

Step 1 – Solve A.30 (2.21) for B_i and A.31 (2.22) for B_0.

$$B_i = \lambda p_i (x_i - \gamma_i)$$ A.32

$$B_0 = \lambda w (l - \gamma_l)$$ A.33

Step 2 – Sum equations A.32 and A.33 to get 2.23. Note that $\sum B_i$ is equal to 1.

$$\sum B_i + B_0 = \lambda \sum p_i (x_i - \gamma_i) + \lambda w (l - \gamma_l)$$ A.34

$$1 = \lambda \left\{ \sum p_i x_i - \sum p_i \gamma_i + w (l - \gamma_l) \right\}$$ A.35

Step 3 – Solve for λ and substitute $[w(T-l)]$ into equation A.35 for $\sum p_i x_i$.

$$\lambda = \left(wT - wl + y - \sum p_i \gamma_i + wl - w\gamma_l\right)^{-1} \quad \text{A.36}$$

$$\lambda = \left(wT + y - \sum p_i \gamma_i - w\gamma_l\right)^{-1} \quad \text{A.37}$$

Equations 2.28 and 2.29

Given

$$\int_0^T e^{-\rho t} U[L(t), X(t)]dt \quad \text{A.39}$$

$$A(0) + \int_0^T e^{-rt}\{W(t)[M - L(t)] - P(t)X(t)dt\} = 0 \quad \text{A.40}$$

Step 1 – Set up the Lagrange and differentiate with respect to time away from work and consumption.

$$\Psi = \int_0^T e^{-\rho t} U[L(t), X(t)]dt + \lambda\left[A(0) + \int_0^T e^{-rt}\{W(t)[M - L(t)] - P(t)X(t)dt\}\right] \quad \text{A.41}$$

$$\frac{\partial \Psi}{\partial L(t)} \Rightarrow \frac{\partial U}{\partial L(t)} e^{-pt} - \lambda e^{-rt} W(t) = 0 \quad \text{A.42}$$

$$\frac{\partial \Psi}{\partial X(t)} \Rightarrow \frac{\partial U}{\partial X(t)} e^{-pt} - \lambda e^{-rt} P(t) = 0 \quad \text{A.43}$$

Step 2 – Define $\frac{\partial U}{\partial L(t)}$ and $\frac{\partial U}{\partial X(t)}$ as U_1 and U_2, respectively, and solve. It is then evident how to get 2.28 and 2.29.

$$U_1(t) = \frac{\lambda e^{-rt} W(t)}{e^{-\rho t}} = \lambda e^{(\rho - r)t} W(t) \quad \text{A.44}$$

$$U_2(t) = \frac{\lambda e^{-rt} P(t)}{e^{-\rho t}} = \lambda e^{(\rho - r)t} P(t) \qquad \text{A.45}$$

Equation 2.32

Given

$$U_1(t) - \lambda e^{(\rho - r)t} W(t) = 0 \qquad \text{A.46}$$

$$U_2(t) - \lambda e^{(\rho - r)t} P(t) = 0 \qquad \text{A.47}$$

$$L(t) = L[\lambda e^{(\rho - r)t} W(t), \lambda e^{(\rho - r)t} P(t)] \qquad \text{A.48}$$

$$X(t) = X[\lambda e^{(\rho - r)t} W(t), \lambda e^{(\rho - r)t} P(t)] \qquad \text{A.49}$$

Step 1 – Substitute the right hand side of equations A.48 and A.49 into equations A.46 and A.47.

$$U_1(t)\{L[\lambda e^{(\rho - r)t} W(t), \lambda e^{(\rho - r)t} P(t)], X[\lambda e^{(\rho - r)t} W(t), \lambda e^{(\rho - r)t} P(t)]\} - \lambda e^{(\rho - r)t} W(t) = 0 \qquad \text{A.50}$$

$$U_2(t)\{L[\lambda e^{(\rho - r)t} W(t), \lambda e^{(\rho - r)t} P(t)], X[\lambda e^{(\rho - r)t} W(t), \lambda e^{(\rho - r)t} P(t)]\} - \lambda e^{(\rho - r)t} P(t) = 0 \qquad \text{A.51}$$

Step 2 – Differentiate equations A.50 and A.51 with respect to W and P from equations A.48 and A.49.

$$(U_{11}L_1 + U_{12}X_1) \;\; (U_{11}L_2 + U_{12}X_2) \;\; (-1) \;\; (0) = 0 \qquad \text{A.52}$$

$$(U_{21}L_1 + U_{22}X_1) \;\; (U_{21}L_2 + U_{22}X_2) \;\; (0) \;\; (-1) = 0 \qquad \text{A.53}$$

$$\begin{bmatrix} (U_{11}L_1 + U_{12}X_1) & (U_{11}L_2 + U_{12}X_2) \\ (U_{21}L_1 + U_{22}X_1) & (U_{21}L_2 + U_{22}X_2) \end{bmatrix} \begin{bmatrix} (-1) & (0) \\ (0) & (-1) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \begin{matrix} \text{A.54} \\ \text{A.55} \end{matrix}$$

<u>Step 3</u> – Take equations A.52 and A.53 and set up a linear representation.

$$\begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix} \begin{bmatrix} L_1 & L_2 \\ X_1 & X_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{A.56}$$

Equation 2.33

Given

$$\begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix} \begin{bmatrix} L_1 & L_2 \\ X_1 & X_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{A.57}$$

<u>Step 1</u> – Multiply both sides of equation A.55 by the inverse of the first matrix on the left, as seen in equation A.56. This yields equation A.57.

$$\begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix}^{-1} \begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix} \begin{bmatrix} L_1 & L_2 \\ X_1 & X_2 \end{bmatrix} = \begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{A.58}$$

$$\begin{bmatrix} L_1 & L_2 \\ X_1 & X_2 \end{bmatrix} = \frac{1}{D} \begin{bmatrix} U_{22} & -U_{12} \\ -U_{21} & U_{11} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{A.59}$$

APPENDIX B: BLUNDELL ESTIMATES

Table C.1[8]

Variable	Estimate	Variable	Estimate	Variable	Estimate
ln c	0.8503 (0.1511) [0.012]	ln c * 1992	0.0037 (0.0056) [0.083]	Family size	0.0272 (0.0090)
ln c * High School dropout	0.0730 (0.0718) [0.050]	ln c *One child	0.0202 (0.0336) [0.150]	ln p_{food}	-0.9784 (0.2160)
ln c * High School graduate	0.0827 (0.0890) [0.027]	ln c *Two children	-0.250 (0.0383) [0.120]	ln $p_{transports}$	5.5376 (8.0500)
ln c * 1981	0.1151 (0.1123) [0.053]	ln c *Three children	0.0087 (0.0340) [0.197]	ln $p_{fuel + utilities}$	-0.6670 (4.7351)
ln c * 1982	0.0630 (0.0387) [0.052]	One child	-0.1568 (0.3215)	ln $p_{alcohol + tobacco}$	-1.8684 (4.1425)
ln c * 1983	0.0508 (0.0704) [0.048]	Two children	0.3214 (0.3650)	Born 1955-59	-0.0385 (0.0554)
ln c * 1984	0.0478 (0.0662) [0.051]	Three children+	0.0132 (0.3259)	Born 1950-54	-0.0085 (0.0477)
ln c * 1985	0.0304 (0.0638) [0.064]	High school dropout	-0.7030 (0.6741)	Born 1945-49	-0.0060 (0.0406)
ln c * 1986	0.0223 (0.0587) [0.068]	High school graduate	-0.8458 (0.8298)	Born 1940-44	-0.0051 (0.0348)
ln c * 1987	0.0528 (0.0599) [0.065]	Age	0.0122 (0.0085)	Born 1935-39	-0.0044 (0.0273)
ln c * 1988	0.0416 (0.0458) [0.049]	Age^2	-0.0001 (0.0001)	Born 1930-34	0.0032 (0.0193)
ln c * 1989	0.0370 (0.0373) [0.046]	Northeast	0.0087 (0.0065)	Born 1925-29	-0.0051 (0.0140)
ln c * 1990	0.0187 (0.0295) [0.060]	Midwest	-0.0213 (0.0105)	White	0.0769 (0.0129)
ln c * 1991	-0.0004 (0.0318) [0.111]	South	-0.0269 (0.0096)	Constant	-0.6404 (0.9266)
OID test				20.92 (d.f. 18; χ^2 p-value 28%)	
Test that income elasticity does not vary over time				27.69 (d.f. 12; χ^2 p-value 0.6%)	

[8] This is an exact reproduction of Table V from Blundell et al. (2006)